ISSN 1075-7007, Studies on Russian Economic Development, 2017, Vol. 28, No. 3, pp. 266–270. © Pleiades Publishing, Ltd., 2017.
Original Russian Text © A.A. Akaev, N.S. Ziyadullaev, A.I. Sarygulov, V.N. Sokolov, 2017.

A Predictive Model of Economic Dynamics during Stagflation Taking into Account the Volatility of the National Currency

A. A. Akaev[a], N. S. Ziyadullaev[b, *], A. I. Sarygulov[c], and V. N. Sokolov[d]

[a] *Institute of Mathematical Studies of Complex Systems, Moscow State University, Moscow, Russia*
[b] *Institute of Market Problems, Russian Academy of Sciences, Moscow, Russia*
[c] *St. Petersburg State University of Architecture and Civil Engineering, St. Petersburg, Russia*
[d] *St. Petersburg State Engineering and Economic University, St. Petersburg, Russia*
**e-mail: nabi926@mail.ru*

Received June 21, 2016

Abstract—This paper considers the mathematical model of economic dynamics under the conditions of stagflation, which was previously developed by the authors and is now generalized for the case of the volatility of the national currency due to the volatility of oil prices. The model is used for the medium-term forecast of economic development in Russia up to 2020.

DOI: 10.1134/S1075700717030029

The model of economic dynamics in the medium term during stagflation, as developed by the authors [1, 2], showed quite good results when verified based on the example of the Russian economy over the past five years. Thus, in the short-term forecast of Russia's economic dynamics of 2015, the rate of decline of the economy was estimated by the model as 4.5% and the inflation rate as 14.5%, while the actual values of these parameters at the end of 2015 were 3.7 and 12.9%, respectively. The refined data of Rosstat on inflation, calculated based on quarterly data, showed complete agreement with the estimation by the model (14.5%).

However, in our opinion, the shortcomings of the model include, first of all, the fact that it does not account for changes in the national currency due to changes in commodity prices, which is characteristic of developing countries. This paper generalizes the model for the case of the volatility of the national currency, as well as provides a regression model that links the ruble exchange rate with the market prices of oil. Thus, we obtain a generalized model that describes the medium-term economic dynamics during stagflation given the volatility of the ruble conditioned by the volatility of oil prices. Based on its use, we calculated the forecast of economic development of Russia up to 2020.

Mathematical model of economic dynamics during stagflation taking into account the volatility of the ruble exchange rate. Let us consider the basic equation and the preconditions of the desired model [1, 2]. The money demand function is described by the Pareto-type distribution function

$$(M/PY)^D = k(r + \pi^e)^{-\alpha}, \ \alpha > 0, \ k = \text{const}, \tag{1}$$

where M is the monetary base, P is the price level in the economy, Y is the real income (GDP), r is the real interest rate, π^e is the expected inflation, and α is the parameter.

The correction of expectations is carried out in accordance with the mechanism of adaptive expectations as follows:

$$\dot{\pi}^e = \beta(\pi - \pi^e), \ \beta > 0, \tag{2}$$

where $\pi = \dot{P}/P$, $\pi^e = \dot{P}^e/P^e$, and β is the coefficient characterizing the rate at which economic agents are reviewing (adjust) their expectations.

Below, it is assumed that *part of the budget deficit d* (as a fraction of GDP) is financed by money emissions as follows:

$$\dot{M}/PY = d^{\xi}, \ \xi > 0, \ d^{\xi}/d < 1. \tag{3}$$

The rest of the budget deficit (d–d^2) can be covered, e.g., as in Russia, at the expense of the Reserve Fund.

Finally, *deviations of output Y from the equilibrium level* $\overline{Y}$, which are caused by the deviation of the price level P from the expected price level P^e, are described by the Lucas supply function:

$$\ln Y - \ln \overline{Y} = b(\ln P - \ln P^e), \ b = \text{const}. \tag{4}$$

As is known, taking into account the exchange rate of the national currency $E(t)$, the nominal interest rate i is calculated by the following formula [3, p. 185]:

$$i = r + \pi^e + \gamma q_E^e, \; q_E^e = \dot{E}^e / E^e, \; \gamma = \text{const}, \quad (5)$$

where E^e is the expected exchange rate of the national currency, e.g., RUR/USD. Consequently, the *function of demand for money* (1) taking into account changes in the national currency (5) can be written as:

$$(M/PY)^D = k(r + \pi^e + \gamma q_E^e)^{-\alpha}. \quad (6)$$

All preconditions of the basic model (2)–(4) shall remain in force. Thus obtained generalized model makes it possible to derive analytical formulas for calculating the predicted trajectories of changes in economic growth (recession) and inflation in the medium term taking into account volatility of the national currency conditioned by volatility of commodity prices, which is characteristic of developing countries.

The conclusion of these formulas is as follows. First of all, we take the logarithmic derivative of both sides of the function of demand for money (6) as follows:

$$\begin{aligned} &\mu - \pi - q_Y \\ &= -\alpha[(\dot{r} + \dot{\pi}^e + \gamma \dot{q}_E^e)/(r + \pi^e \gamma q_E^e)], \end{aligned} \quad (7)$$

where $\mu = \dot{M}/M$; $\pi = \dot{P}/P$; $q_Y = \dot{Y}/Y$.

Since, in the retrospective zone $\pi^e = \pi$ and $q_E^e = q_E$, Eqs. (6) and (7) can be used to estimate the values of parameters k, γ, and α. Using the actual data of Rosstat for key variables (M, π, q, γ, r, and E) for 2011–2015, for the Russian economy, we obtained the following estimates of the parameters: $k = 70.0$, $\gamma = -398533$, and $\alpha = 0.32$.

Next, we consider Eq. (3), which describes the financing of the partial budget deficit through money emission. We transform the left-hand side of this equation using expression (6) for the money-demand function as follows:

$$\begin{aligned} \dot{M}/PY &= (\dot{M}/M)(M/PY) \\ &= \mu k(r + \pi^e + \gamma q_E^e)^{-\alpha}. \end{aligned} \quad (8)$$

Then, Eq. (3) can be written as

$$k\mu(r + \pi^e + \gamma q_E^e)^{-\alpha} = d^{\xi}. \quad (9)$$

The substitution of the logarithmic derivative of both sides of Eq. (9), i.e.,

$$\begin{aligned} &-\alpha[(\dot{r} + \dot{\pi}^e + \gamma \dot{q}_E^e)/(r + \pi^e \gamma q_E^e)] \\ &= \xi(\dot{d}/d) - \dot{\mu}/\mu, \end{aligned} \quad (10)$$

to the right side of Eq. (7) yields the *key equation of the model*

$$\pi + q_Y = \mu + \dot{\mu}/\mu - \xi(\dot{d}/d). \quad (11)$$

As can be seen from this equation, during stagflation, in the medium term, the economic dynamics is completely determined by two factors, i.e., the rate of monetary growth (μ) and budget deficit (d).

In order to separate two key variables π and q_y of our interest in Eq. (11), we turn to the supply equation of Lucas (4). Let us differentiate both the sides. The result is

$$q_Y = q_{\bar{Y}} + b(\pi - \pi^e). \quad (12)$$

We use Eq. (2) to review the expectations and, in the right-hand side of Eq. (12), we replace $\dot{\pi}^e/\beta$ with $\pi - \pi^e$. As a result, we obtain the following equation:

$$q_Y = q_{\bar{Y}} + \rho\dot{\pi}^e, \; \rho = b/\beta. \quad (13)$$

Since, in the retrospective zone, $\dot{\pi}^e = \dot{\pi}$, this equation can be used to estimate values of parameters q_Y and ρ. Using the actual values of π and q_Y (Rosstat monthly data for 2014–2015), we obtain the following parameters: $q_Y = 0.021$ and $\rho = 2.3 \times 10^{-7}$. Thus, according to our estimates, the equilibrium potential level of economic growth in the medium term is only 2.1% ($q_Y = 0.021$). This level is extremely insufficient for the Russian economy transition to a higher trajectory of dynamic growth, so it is necessary to search for new sources of growth.

The expression for π^e is obtained from Eq. (9) as follows:

$$\pi^e = (k\mu/d^{\xi})^{1/\alpha} - r - \gamma q_E^e. \quad (14)$$

By substituting the derivative of expression (14)

$$\begin{aligned} \dot{\pi}^e &= 1/\alpha(k\mu/d^{\xi})^{1/\alpha} \\ &\times[\dot{\mu}/\mu - \xi(\dot{d}/d)] - \dot{r} - \gamma\dot{q}_E^e. \end{aligned} \quad (15)$$

into Eq. (13), we obtain the *final formula for predictive calculations of the dynamics of economic growth* (recession) as follows:

$$\begin{aligned} q_Y &= q_{\bar{Y}} + \rho[1/\alpha(k\mu/d\xi)^{1/\alpha} \\ &\times(\dot{\mu}/\mu - \xi\dot{d}/d) - \dot{r} - \gamma\dot{q}_E^e]. \end{aligned} \quad (16)$$

First of all, this formula is used to estimate the value of parameter ξ based on actual data of all variables for the years 2014–2015 (using monthly or quarterly data). The obtained estimate is $\xi = 0.02$. Knowing the predictive dynamics q_Y, from (11), we find the *formula for calculating the predictive dynamics of inflation* as follows:

$$\pi = \mu + \dot{\mu}/\mu - \xi(\dot{d}/d) - q_Y. \quad (17)$$

This equation clearly shows that, among other factors, economic growth helps to reduce inflation.

Strategies for changing the control variables of the model and modeling economic growth rates. As input data, predictive calculations using formulas (16) and

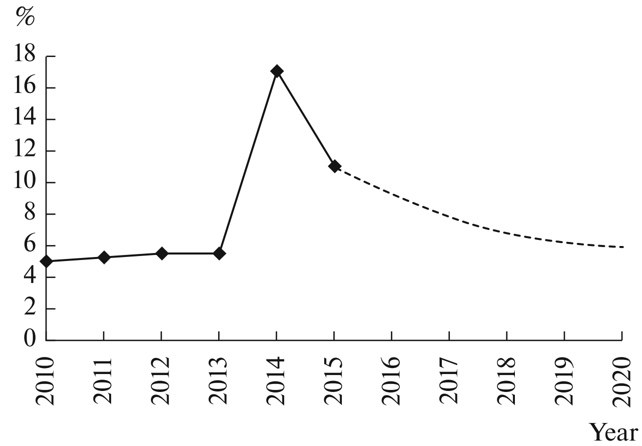

Fig. 1. Key rate at the end of the year (up to September 13, 2013, the interest rate on repo transactions): –◆– fact; ---- forecast.

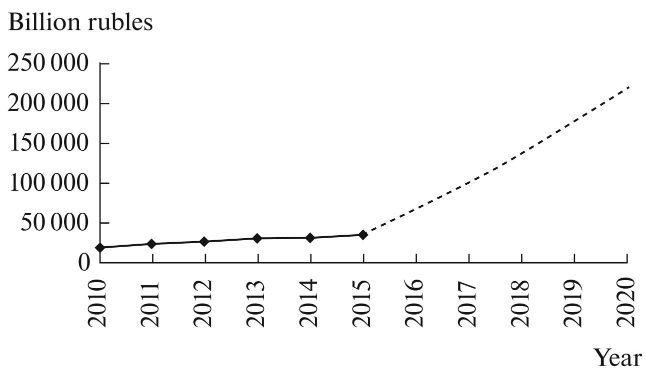

Fig. 2. Money supply M_2 (at the end of the year): –◆– fact; ---- forecast.

(17) require strategies for expanding (compressing) the money supply (μ), decreasing (increase) the budget deficit (d), and reducing the key interest rate (r), as well as the data about the expected trajectory of the national currency exchange rate (E). Thus, control variables are the key interest rate (r), rates of monetary growth (μ), and the size of the budget deficit (d). It is obvious that, when forecasting using models (16) and (17), it may become important to search for strategies of changing the control variables that would lead to the early exit of the economy from the recession and the achievement in the medium term (preferably in 2020) of the potential equilibrium rates of economic growth $q_{\bar{Y}} = 2.1\%$ set forth above.

Key interest rate. The strategy for reducing the key interest rate r is expedient to be formulated as the gradual decline to the equilibrium level of 6% [2] by the end of 2019 to the beginning of 2020 (Fig. 1) [7]. This strategy can be described by a simple polynomial of the second degree as follows:

$$r = r_1 + r_0(T_1 - t)^2, \ \dot{r} = -2r_0(T_1 - t), \quad (18)$$

where $T_1 = 2020$, $r_1 = 0.06$, and $r_0 = 0.002$.

This strategy assumes that, by early 2017, the key rate will drop to 9.2%. This will help to reduce the key interest rates in June 2016 to 10.5%.

M_2 money supply. To describe the money supply, we also propose a square law as follows:

$$\begin{gathered} M = c_0 + c_1(t - T_0) + c_2(t - T_0)^2, \\ \dot{M} = c_1 + 2c_2(t - T_0), \ \ddot{M} = 2c_2, \ \mu = \dot{M}/M, \\ \dot{\mu} = \ddot{M}/M - (\dot{M}/M)^2 = 2c_2/M - \mu^2, \end{gathered} \quad (19)$$

where $T_0 = 2015$ and c_0, c_1, c_2 are coefficients.

Calculations of the rate of economic growth by formula (16) showed that, as shown in Fig. 2 [4], coming out of the recession and achieving potential equilibrium growth rates of 2.1% require an increase in money supply growth (μ) by two to three times compared to their current values. For the best trajectory of monetary growth that would make it possible to achieve the above goal provided that by 2020 inflation is progressively reduced to 5%, we obtained the following estimates of the coefficients of predictive function (19): $c_0 = 35\,000$, $c_1 = 30\,000$, $c_2 = 1400$.

Budget deficit. In 2015, the budget deficit was 3.5% of the GDP. In 2016, the deficit was planned to be 3%, which should be financed mainly by means of the Reserve Fund. It was supposed to spend 2.5 trillion rubles for this purpose. In 2017, the Ministry of Finance again planned the deficit to be 3% of the GDP; then it will be reduced by 1% per year to 1% of the GDP by 2020. However, this goal may be unattainable due to sluggish economic growth. In any case, given that there has been a sharp increase in the deficit from zero to 3.5% of the GDP in just 4 years (2012–2015), and its decline will be long and difficult, it is advisable to describe the dynamics of this indicator by the following function:

$$\begin{gathered} d = \delta(t - T_2)^{3/2} e^{-\lambda(t - T_2)^2}, \\ \dot{d} = \delta(t - T_2) e^{-\lambda(t - T_2)^2} [3/2 - 2\lambda(t - T_2)^2], \end{gathered} \quad (20)$$

where $T_2 = 2011$ is the last year with a surplus budget, δ and λ are parameters.

Modeling economic growth by formula (16) taking into account the chosen strategy to reduce the budget deficit (20) showed that, in order to achieve the potential equilibrium economic growth rates by 2020, the deficit may be reduced to only 2% of the GDP from the current 3.5% (Fig. 3) [5]. At the same time, the following estimates for the constant parameters were obtained: $\delta = 0.0065$, $\lambda = 0.028$.

Ruble exchange rate. Some experts believe that the ruble started free floating and no longer depends on oil prices. However, this is not true. Oil remains the determining factor in the ruble exchange rate. Most experts say that the ruble is still entirely dependent on oil prices. However, the extent of this relationship is

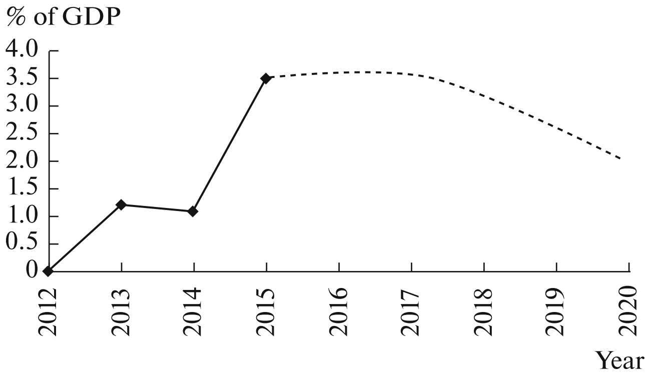

Fig. 3. Deficit of the consolidated budget of the Russian Federation: —◆— fact; ---- forecast.

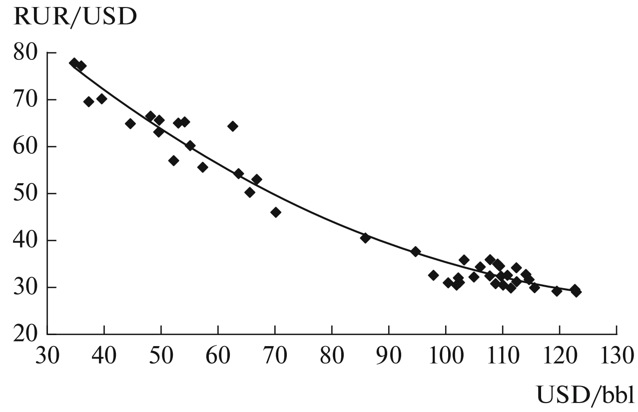

Fig. 4. Correlation field for oil prices and the dollar exchange rate ($R^2 = 0.972$).

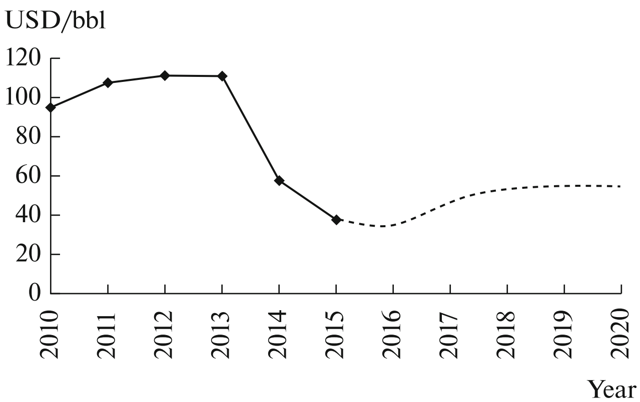

Fig. 5. Oil prices (at the end of the year): —◆— fact; ---- forecast.

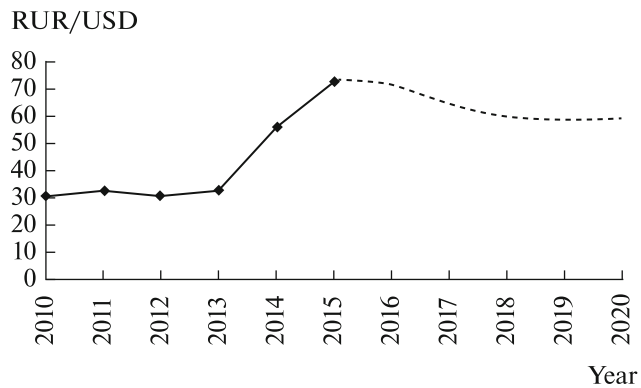

Fig. 6. Dollar exchange rate (at the end of the year): —◆— fact; ---- forecast.

determined by the level of oil prices; at low oil prices, there is 100% dependence and, at high oil prices, the dependence is about 75% or higher. Thus, at low oil prices, the statistical significance of the correlation of oil prices and the ruble exchange rate is very high, but it decreases with an increase in oil prices.

In order to establish an analytical link between the current prices of Brent crude oil P_{OB} (USD/bbl) and the ruble exchange rate E (RUR/USD), we use the simple quadratic regression

$$\begin{gathered} E = a + bP_{\mathrm{OB}} + cP_{\mathrm{OB}}^2, \\ \dot{E} = \dot{P}_{\mathrm{OB}}(b + 2cP_{\mathrm{OB}}),\ q_E = \dot{E}/E. \end{gathered} \tag{21}$$

Using the data on the prices of Brent crude oil and the ruble exchange rate over the period of 2012–2016, we obtained the following values for the regression coefficients: $a = 100.2$; $b = -0.914$, and $c = 0.0028$.

The resulting regression curve (21) and the actual correlation field of oil prices and the ruble exchange rate are shown in Fig. 4 [4, 6]. As can be seen, there is a strong correlation ($R_2 = 0.97$).

Predicting oil prices. Let us make a medium-term forecast for Brent crude oil prices. As is known, the average annual price of Brent crude oil in 2015 was 52 USD/bbl. However, in early 2016, oil prices fell to 30 USD/bbl, but then quickly recovered and already in the beginning of June 2016 again overcame the level of 50 USD/bbl. For the next period, most experts forecast a corridor of 45–55 USD/bbl. The most farsighted experts argue that 50 USD/bbl is the equilibrium price in the long run due to the cost of shale oil in the United States, which plays a key role in the supply–demand ratio, is 40–50 USD/bbl. The decline in oil prices was due to its overproduction in recent years on one hand and the strengthening of the US dollar on the other hand. Since the US dollar has entered into a long cycle of appreciation, which will last at least until 2025, here comes the same prolonged period of moderate or low oil prices. Therefore, a further substantial increase in oil prices should not be expected.

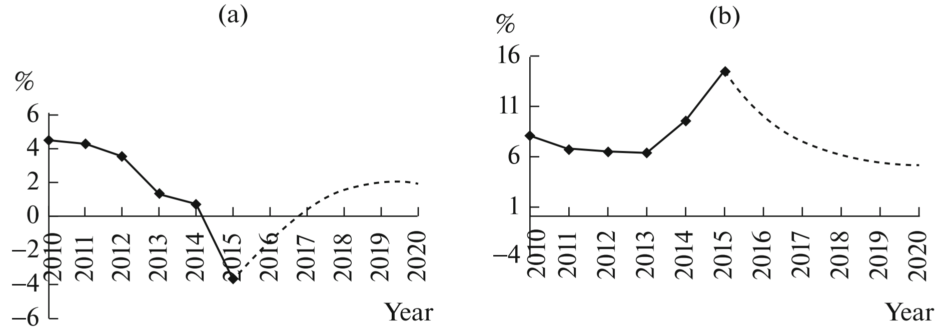

Fig. 7. Forecast of (a) GDP growth and (b) inflation for Russia until 2020: –◆– fact; ---- forecast.

Based on the above considerations, as the predictive trajectory of rising oil prices, we selected a logistic curve with a saturation level of 55 USD/bbl by 2020:

$$P_{\mathrm{OB}} = P_{\mathrm{OB}}^{(0)} + \frac{P_{\mathrm{OB}}^{(1)}\{1 - \exp[-\vartheta(t - T_0)]\}}{1 + P_{\mathrm{OB}}^{(1)}\exp[-\vartheta(t - T_0)]};$$
$$\dot{P}_{\mathrm{OB}} = \frac{\vartheta(1 + P_{\mathrm{OB}}^{(1)})P_{\mathrm{OB}}^{(1)}\exp[-\vartheta(t - T_0)]}{\{1 + P_{\mathrm{OB}}^{(1)}\exp[-\vartheta(t - T_0)]\}^2}, \tag{22}$$

where $T_0 = 2015$; $P_{\mathrm{OB}}^{(0)} = 30$; $P_{\mathrm{OB}}^{(1)} = 25$; $\vartheta = 2$.

The forecast trajectory of oil prices movement is shown in Fig. 5 [6] by a dotted logistic curve. The corresponding forecast trajectory of the ruble exchange rate calculated using formula (21) is shown in Fig. 6 [4]. As can be seen, the ruble exchange rate will strengthen and stabilize at 60 RUR/USD.

Forecast of the medium-term dynamics of economic growth and inflation. The final results of predictive calculations of economic growth and inflation rates up to 2020 calculated according to formulas (16) and (17) shown in Fig. 7 [7] indicate that, in 2016, the recession will amount to −1.4% (Fig. 7a); in 2017, a slight increase is expected of only 0.4%; in 2018, significant growth of 1.6% is expected, then in 2020 followed by up to 2%.

According to our forecasts, inflation is above the target indicators set by the Central Bank of Russia and will amount to 9.9% in 2016 and 7.6% in 2017; it will only decrease to 5% in 2020 (Fig. 7b). Of course, the Central Bank can achieve the intended targeting goals to achieve a 4% inflation rate by early 2018, but only at the cost of continued stagnation in the economy, since it would require further contraction of the money supply. As noted above (see Fig. 2), ensuring speedy recovery from the recession and achieving a potential equilibrium growth rate of 2.1% by 2020 requires increasing the rate of monetary growth by two to three times.

ACKNOWLEDGMENTS

The paper was prepared with the financial support of the grant of the Russian Scientific Fund (project no. 14-28-00065) Structural and Cyclical Paradigm of Economic and Technological Renewal of Macro-Social Systems (World and Russia in the first half of the 20th century).

REFERENCES

1. A. A. Akaev, A. I. Sarygulov, and V. N. Sokolov, "Mathematical models of economic dynamics in the context of high inflation and unstable development," Dokl. Math. **92** (3), 757–760 (2015).
2. A. A. Akaev, N. S. Ziyadullaev, A. I. Sarygulov, and V. N. Sokolov, "Medium-term forecast of the dynamics of the development of the Russian economy," Stud. Russ. Econ. Dev. **27** (5), 510–517 (2016).
3. B. E. Brodskii, *Macroeconomics* (Magistr, Moscow, 2012) [in Russian].
4. http://www.cbr.ru/.
5. http://minfin.ru/ru/statistics/conbud/index.php#.
6. http://www.investing.com/commodities/brent-oil-historical-data.
7. http://www.gks.ru/wps/wcm/connect/rosstat_main/rosstat/ru/statistics.

Translated by K. Lazarev