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The Agent-Based Model of the Closed Market with One Commodity and with two mechanisms (with different degree of risk) of individual behavior of participants of market [1]

The model of market of one commodity , in which there are in each moment of time the same quantity and the same quantity of money was formulated and researched in this paper. Each partner of the market in the one moment of time can be in one of three status: to be buyer, be seller and do not take part in trade in this moment of time. In addition each of them can change his status in the next moment of time. Partners of market change their statuses and prices, by using the personal information of each of them about trade in the previous moment of time only. The main target of this research is the searching of effects of rational behavior of partners of the market and difference of trajectories at different degree of risk in the choice of actions. Some characteristics of dynamics of average price of market in the case of careful choice only or in the case of one variant of risky choice only were received as a result of our research by computer model. The nature of dynamics of the set of prices of participants was investigated analytically. The main result is the convergence of trajectory of our system to stationary set of states with average price of trade which is close to some constant when behavior of all agent is careful and bounded hesitation of this trajectory when there are risky agents only. These facts are established by series of experiments with computer realization of the model. The behavior of trajectory of system was investigated by computer experiments in the case when all agents are identical simple determinate automata with linear tactic with careful and risky actions..

.Keywords: mathematical model, closed market, one commodity market, dynamics of prices, trajectory, stationary set, steady state, rational choice.

JEL Classification: c51, D01.

1. Introduction

There are markets where any partner can be buyer at the one moment of times, he can be seller in other moment and not take part in trade in the third moment. These markets play very important role in the economic mechanism now. The stock markets are one example of these markets. Several logical connected chains of exchanges are directed to receiving of profit. The serious role of these markets in

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economic mechanism is a principal reason of interest to research of these markets. Most interest is the research of dynamics of such system and description of neighborhood of equilibrium or steady states.

Richard Topol's paper [3] the most likely was a first research where model of financial market with one asset in which behavior of autonomous participants of market of assets was simulated by stochastic process had been considered. Each partner can be seller or buyer which is characterized in this moment of time by his own price (of selling or of purchase) in dependence of his position on the market. The interaction between partners represents itself by trade. The author supposes that the two partners can meet each other and they both are agree some price of the trade.

The analysis of model showed that some probability distribution at which the herd behavior is most likely, is a stationary for stochastic process which represents the dynamics of this model.

The approach to researching of dynamics of market which bases on formal description algorithm of decision making by individual partner of market and description of mechanism of interacting of partners, with following studing of trajectory of the system, steady states and stationary sets of states is most promising by our opinion. This approach is in frames of the more common theory - theory agent based models, which is represented in works V.L.Makarov ,,A.R.Baxtizin and Alan Kirman [4],[5],[6]1.

The very interesting agent based model, which known as Santa Fe model of Artificial Sock Market model was created by Blake Le Baron, W Brian Arthur and Richard Palmer [7]. But this model is very difficult for analytical research because algorithms of behavior and interaction of participants imitate the character of real behavior on the stock market too exactly, may be, but this model demonstrates of a dynamics of the market which are connected with process of learning participants and individual estimation the current market state by agents., The sequential research of models the complications of which is increasing sequentially may be beginning with those which demonstrate principal features of market mechanism is a perspective approach by our opinion. The works of famous mathematicians Gelfand, Tsetlin and their coworkers [1],[2] in which they has investigated collective behavior of automata is source of our approach to studing of market models. Ideas of these works were used in the formulation and research of model of nonclosed market with one commodity where a market was considered as a system of interacting automata. Author be able to prove that trajectory of system reaches the neighborhood of steady state of system (equilibrium of demand and supply) from any initial state.[8] To create such model of closed market which should   repel the   short-term and realistic situation on the stock market was not easy for us. Nevertheless, finally we could formulate the agent-based model of the closed market with one commodity   i.e. model of market, in which there are the same quantity of commodity and the same quantity of money in each moment of time [9],[10]. Participant of the market can be seller, buyer or not take part in trade in each moment of time. But in next moment of time each participant can change his status: i.e. a seller can become the buyer or the partner which is waiting. There is the same situation for a buyer and for a waiting partner. Participant of trade change their status and declare new prices by using of his own information only . In each moment of time the buyer which has a money and agrees to pay the maximal price is trading with the seller which has a commodity and agrees to receive the minimal price . Moreover the prognosis of average price in the next moment sometimes stimulates sellers to change their status , i.e. to become buyers in the next moment or to refuse to take part in trade for several moments of time (to be waiting). The same take place for a buyer. The simple algorithm of changing by participant his status(seller, buyer, waiting) in the next moment of time was formulated in the papers. Moreover algorithm of decision making about price and status in the next moment of time i.e. decision which participant make in the given moment of time includes in herself only logically justified actions at assumption of realization of prognosis of average price of exchange. In addition agent uses only information about result of trade in previous moment and only part of information about state of all market, namely the average price of a commodity for the previous moment of time. Different variants of simple algorithms could be considered: rational and irrational choice of prices and statuses. We have not considered exactly irrational choice in these stages of our research , but we considered a choice with the some risk and very careful choice. Some hypothesizes about character of behavior of trajectory of system were confirmed in these papers. We could receive some analytical results about dynamics of set of prices of the model of market but we could not receive all analytical properties of stationary trajectory. The mathematical model was represented as a computer program and we have investigated by computer modeling some characteristics of simplest closed market. The main result of this research is the formal model of interaction of participants and mechanism of decision making by participants. In the last part of the research we studied by the computer model more complicated behavior of participant of market at choice of price. The one orientation of choice was in model before to sell a commodity by most possible price and to buy by lest possible price. We consider the simplest automata as a participant of market with two action and memory and expedient behavior. The goal of these automata is to receive maximal possible amount the money including the cost of his commodity.

The simple model of closed market with single commodity has formulated in the second section of the paper. Each participant of market is characterized by quantities of commodity and money, by his price, his status (seller, buyer, waiting agent) and his relation to the risk. The last value equals a unit when the given agent in this moment of time prefers the careful action and this value equals minus unit when the given agent in this moment of time prefers the action which induces more risk. In addition if the agent is a seller then his price means the price with which he agree to sell his commodity, but if he is a buyer then his price means the price which he agree to pay for commodity. At last if the agent take no part in trade on this market (waiting), then his price is used as a orientation for change of a status i.e. he compares his price and prognosis of average market price and he make decision the following: to choose a status of seller or buyer or to remain waiting the situation which corresponds to one of such transformations. We suppose that each moment consists from two tacts (steps of time). Also we suppose that each seller knows all prices, which buyers offer in this moment and each buyer knows all prices which sellers ask in this moment. Each seller offers all quantity of commodity which he has and each buyer is ready to pay all his money to buy commodity. Sellers, which ask the minimal price, sell their commodity to buyers which offer maximal price. The exchange the commodity on the money occurs by the price which equals the half-sum of these prices (a seller and buyer have the equal forces in the bargaining) . One of two result is possible: sellers have sold all their commodity or buyers have spent all their money. In the first case buyers, which still have money after this trade, ask the commodity from sellers with the price with the next value(more than before). In the second case sellers, which have commodity still after last step of trade, propose their commodity to buyers with the price with next value (less than before). The process is finished in two cases. The first when there is no price, offered by potential buyers, which is more than minimal price of sellers which still have commodity. The second, when there is no price requested by potential seller, which is less than maximal price of buyers which still have money. Each participant of exchange receive own information about result of exchange after finish of exchange: quantity of sold commodity, quantity of money obtained for the commodity in the case of seller and quantity of the bought commodity and quantity of the money spent for his purchase in the case of buyer. Occurring transactions as though are registered. The average price for all transactions for previous moment is communicated to all participants. For simplification of the model (to reduce the effect of prognosis) also was supposed that average price of market in the given moment is a orientation for decision making of participants including waiting participants. Directly it is assumed that participants think the average price in the next moment is close to average price of given moment of time. We use this supposition with the alone goal to simplify the model on the initial stage of the investigation. On the second step of this moment of time each participant make decision about his status in the next moment of time in accordance with it changes his price. He uses is own information and average price of market only in this decision making. The algorithm of choice is maximal simple and the choice is logically justified and it is coordinated with prognosis of average price of market. If the agent took part in trade in the first step this choice is fulfilled on the base of his own information about relation of average price of all his bargains and prognosis of the average price of market. If seller(buyer) realized only part of his commodity(money) or could not take part in the trade, in consequence of relation of prices, or hr was a waiting participant, then he defines his price in the next moment of time By using his price and prognosis of average price of the market(the same for all participants). The same process repeats in the next moment of time on base of results of trade in this moment of time.

The character of dynamics of the set of prices of participants was investigated in the third section. We name this set by spectrum of prices. It is proved that structure of spectrum of prices can be described adequately by several indexes of this set beginning with the some moment of time. The divergence of spectrum of prices i.e. difference between minimal price of buyers and maximal price of sellers, as well the width of spectrum of prices i.e. difference between maximal price of buyers and minimal price of sellers as well some other indexes of the sets of prices of sellers, of buyers and of waiting agents are such several indexes which give to us adequate description of set of price this system. It is proved that these indexes of the set of prices become bounded by some values beginning with some moment of time. The width of spectrum of prices always will be bounded by some constant beginning with some moment of time and divergence of spectrum of prices will be not less than zero.

The results of investigation of our system by series of experiments with computer model are stating in fourth section. We investigated the case when all participants are careful agents in the first part of section , after in we considered the case when all participants are risky agents and case w hen part of participants are careful agents and other participants are risky agents. The central result of this section consists in the fact that after some moment of time τ_p the average price of our market changes inside of the some interval. If we have calculated the value of this average price averaged on time during the time from $\tau_p$until very large time (more than 10000 moments) then this value of the average price averaged on time will be inside this interval of prices. This interval is rather small in the case when the all participants use careful choice . But this interval is large in the opposite case when all participants are risky agents. In this case the trajectory of average price of the market hesitated irregularly with not constant large periods. In the case when the one part of participants are risky agents and other participants are careful agents there was a intermediate situation. Interval was less than in the case of all risky participants and more than in the case of all careful participants. The same was in relation to periods of hesitations.

In the fifth section we consider our agent based model of closed market as a some game of automata (see [1],[2]. We consider the participants which can choose between careful or risky establishment of their price in moment t+1, as a result of their success or not success and their actions before. So our modes presents some game. When we present the mechanism of a choice of price by agents as some finite automaton, we can follow to Gelfand and Tzetlin approach to investigation of games of automata. We modeled the participants of market as very simple determinate automata $L_{2m,2}$ -asymptotically optimal automata with linear tactic and with two possible action and volume of memory m We investigate our system by computer experiments in this section . The assertion about convergence of trajectory of average market prices to some set nearly one value of price take place at our presuppositions only. We can see from many experiments the role of memory of our automaton also.

We discuss our results and problem of modeling of closed market and say about possibility of further research of similar system in the conclusion.

2. The model.

We shall consider the dynamic model of closed market of one commodity with n interacting participants. Closure of the market means that in each moment of time the sum of money which all participants have is equal unit and sum of quantities of commodity which all participants have is equal unit also. The time is proposed discrete: t=0,1,2,…. Participants of market are numbered by i:(i=1,2,3,...,.) the each participant of market has one of three statuses that means that in each moment of time there is the number $\alpha_i(t)$, which can be 1,-1,0 . Each agent is able to have only one from three statuses i.e. $\alpha_i(t) = 1$ indicates that the agent is the seller in this moment, $\alpha_i(t)=-1$ indicates that the agent is the buyer and $\alpha_i(t)= 0$ indicates that this agent takes no part in the trade in this moment of time(he waits the corresponding situation to take part in trade) . Each participant of the model can to have commodity and money simultaneously. It is difference of this model from our model of nonclosed market which we have investigated many years ago. Let denote by $x_i(t)$ the quantity of commodity which agent i has and denote by $y_i(t)$ the quantity of money which agent i has in moment of time t. The price $v_i(t)$ also is characteristic of state of agent i in moment t. When this agent is a seller ($\alpha_i(t) = 1$) he shall not agree in this moment of time t to sell his commodity by price which less that his price $v_i(t)$. When this agent is a buyer ($\alpha_i(t) = 1$) he shall not agree in this moment of time t to buy the commodity by price which more that his price $v_i(t)$. When $\alpha_i(t) = 0$ (agent is waiting) the price of this agent has meaning of the indicator for choice of one from three possible decision of this agent in moment t: will become seller, will become buyer or to remain the waiting. When the participant of market is a waiting agent he can change his price depending on the relation of his price and average price of market in moment of time t which we denote by u(t). Moreover agent has one additional simple variable $k_i(t)(k_i(t) = +1, -1)$ corresponds to agent i in moment of time t. If $k_i(t) = 1$ then agent i changes his price more carefully than in the case $k_i(t+1) = -1$ More exactly it will be defined below. So the state of our model of market in moment of time t is described by the 5N variables . We shall suppose that each moment of time consists from two steps (tacts of time). During the first step takes place the following. Each seller (participant for which $\alpha_i(t) = 1$) proposes to all buyers to buy all his commodities. Just the same way each buyer is ready to spend for purchase of the commodity all his money.

The exchange consists from bargains and consequences of these bargains are defined by relations of prices of sellers and prices of buyers. Let $\alpha_i(t) = 1$ at $i = i_1, i_2, i_3, \dots, i_k$. and let $v_{i_1} \leq v_{i_2} \leq v_{i_3} \leq \dots \leq v_{i_k}$ and also $\alpha_i(t) = -1$ at only $j = j_1, j_2, j_3, \dots, j_l$ and let $v_{j_1} \geq v_{j_2} \geq v_{j_3} \geq \dots \geq v_{j_l}$, So the first bargain happens between seller with minimal price and buyer with minimal price, the price of their trade equals half of sum of two these prices: $(v_{i_1} + v_{j_1})/2$.

Let for definiteness the buyer with number $j_1$used up the all his money for buy of the part of quantity of commodity which seller i_1 had, but some part of commodity remains at seller i_1 after bargain . Then the seller i_1 offer his remainder of commodity to buyer with number j_2 and bargain between them happens in just the same way as was described above. The bargain between them will be fulfilled by price $(v_{i_1} + v_{j_2})/2$.

In the contrary case when after first bargain the seller i_1 sold all his commodity but the some money remained at buyer j_1 after first bargain then buyer j_1 ask the commodity from seller i_2 and bargain between them will be fulfiled by price $(v_{i_2}+v_{j_1})/2$. Further it will the next bargain depending on result of this second bargain. The next bargain will be between seller i_1 and buyer j_3 or between seller i_2 and buyer j_2 (similarly between buyer j_1 and seller i_2 , or between buyer j_2 and seller i_2). The quantity of commodity is decreased and quantity of money increased at sellers and quantity of money is decreased and quantity of commodity increased at buyer, when both take part in bargain.(the variables $x_i(t)$and y_i(t) changing). Such process of sequential bargains will be continued as long as the at least will be fulfilled one from following three conditions. The first: all sellers have no commodity . The second: all buyers have no money. The third: the price of seller who still has a commodity is more than price of buyer who still has a money. We have not considered until now the case when several sellers have in given moment of time the same prices and also the case when several buyers have in given moment of time the same prices moreover both cases can happen simultaneously. In these cases the exchange the commodity on the money take place between one generalized seller with given price of a selling and one generalized buyer with given price of a purchase. After bargain the all money which the generalized seller has received (if he had sold all his commodity) or all commodity which had been bought by generalized buyer (if he had spent all his moneys) distribute between all sellers with given price or between all buyers with given price. . We use in this investigation the following principle of distribution. The sellers with the same price which have small quantity of commodity sold all their commodity other sellers with this price sold only part of their quantity of commodity. The same relate to buyers. That is reason of the result that those sellers and buyers will have the different prices in the next moment of time. Let there are k sellers with the same prices: $v_{i_1}(t) = v_{i_2}(t)$=v_{i_3} (t)=…_=v_{i_k} $(t) = v(t)$ and different $x_{i_1}(t) \geq x_{i_2}(t) \geq x_{i_3}(t) \geq \cdots \geq x_{i_k}(t)$(t).quantities of commodity:. Denote: $X(v,t) = \sum_{s=a}^{s-k} x_{i_s}(t)$ .These sellers trades with buyer (may be also a generalized) who propose the price V_0(t) for commodity and has some quantity of money Y_0(t).Denote In this case not all sellers can to sell all their commodity and in this case there exist such and that.$X(v,t)(v(t) + V_0$(t))/2>Y_0(t). The not all sellers can sell whole his commodity, but all buyers will spend his money.

And the such $l < k$ and H(t) exist that in this case that:$[x_i \ (t) + x_{i_{k-1}}(t) + x_{i_{k-2}}(t) + \cdots + x_{ik-r} \ (t) + (k-r)H(t)](v(t) + V_0(t) = Y_0(t)$

$$x_{i_k}(t) > H(t), x_{i_{k-1}}(t) > H(t), x_{i_{k-2}}(t) > H(t), \ldots, x_{i_{k-r}}(t) > H(t)$$

Thus k-r sellers sell the all his commodity but the rest realized only part of his commodity (moreover the same quantities). In the case of this principle of distribution the sellers which ask the same price are divided on two groups: sellers which sold all his commodity and sellers which sold only part of his commodity. Distribution of expenditures among buyers with the same price happens analogously. Quantity of commodity and quantity of money of agent i which took part in exchange vary once end of the first tact (step of time) of moment of time t is reached and at $\alpha_i(t) = 1$will be x_i(t+1)≤ $x_i(t), y_i$(t+1)≥ y_i(t) and at $\alpha_i(t) = -1$ will be $x_i(t+1) \geq x_i(t), y_i$(t+1)≤ $y_i(t)$. If agent i is a seller and he not took part in trade because his price is rather high for buyers, then quantities of his commodity and of his money not change. If agent i is a buyer and he took not part in trade because his price is rather low for sellers, then quantities of his commodity and of his money not change also. If agent is a waiting agent then it is obviously that quantities of his commodity and of his money not change . We can define the average price of exchanges $w_i(t)$in which this seller or this buyer took part in the first tact(step of time) of moment of time t:

w_i(t)=(y_i(t+1)-y_i(t))/(x_i(t)-x_i(t+1)) if $\alpha_i(t) = 1$

w_i(t)=(y_i(t)-y_i(t+1))/(x_i(t+1)-x_i(t)) if $\alpha_i(t) = -1$

It is possible to define general variables of exchange in this tact and if any exchange happens then we can define average price of trade of whole system at this moment of time.

$\Delta Y(t) = \sum_{i=1}^{i=N}(y_i(t) - y_i\,(t)\,\alpha_i(\mathrm{t})(\alpha_i(t) - 1)/2$

$\Delta X(\mathrm{t}) = \sum_{i=1}^{i=n}(x_i\,(t) - x_i(\mathrm{t+1})a_i(\mathrm{t})(\alpha_i(t) + 1)/2$

if turn out $\Delta X(t) > 0$to be then we set that:

$$u(t) = \frac{\Delta Y(t)}{\Delta X(t)}$$

Where we shall name u(t) the average price of market in the moment of time t.. We shall suppose that center (operator) one for all market exists. All bargains at market register by this center/ He also calculates the average price and declares this average price to all participants of market. The value of average price in moment of time t is a single external information for participant of market. Other information of participant is the his own information(with index i).

In the course of the second tact of moment of time t each participant of market changes his status and after it he determines his new prices. He do it by using the result of trade on the first tact of this moment of time. We are constrained to propose for the simplification of model that participant can use for establishment of new status and new price only information about result of his trade on the first tact of moment t and the average price in moment t. Moreover to make the algorithm of choice of statuses and prices maximal simple we suppose that participants are hoping for that the average price in moment t+1 will be the same that average price in moment t.

Therefore we shall presuppose that agent i chooses his status in moment t+1 in the any situation (result of trade) after first step of moment of time t by means of the comparison of the average price on the market u(t) with average price of his bargains on the first tact of moment t ($w_i(t)$)) . We shall presuppose for simplification of our model that careful agent and risky agent do it by independently from their character. But their characters (careful or risky) will be appear when they will know their statuses already and they will be defining their new prices after it. We proposed the following (rather simple) algorithm for this choice of new statuses. From the beginning we shall consider the logic of seller's decision making about changing of status ($\alpha_i(t) = 1$) . If in the first tact of moment t the seller has sold all quantity of commodity which he has had then in moment t+1 he can become buyer or waiting agent only(he has no commodity) . It is profitable for him to become the buyer if he can hope to buy the commodity in next moment (t+1) more cheaply than he has sold his commodity in moment t. We can propose at the condition of his limited information that agent makes decision in accordance to the following algorithm . If $w_i(t) > u(t)$ he will become the buyer ($\alpha_i(t) = -1$). In other case ($w_i(t) \leq u(t)$) he will became waiting agent ($\alpha_i(t) = 0$). If in the first tact of moment t the seller i (α_i(t)=1)sold only part of his commodity or not took part in trade($x_i(t+1) > 0$) then there is one reason of it. This reason is his price($v_i(t)$) which is rather high for all buyers in this moment. If he has commodity and has money($x_i(t+1) > 0, y_i(t+1) > 0$) then he can choose to become a buyer($\alpha_i(t+1) = -1$) or a seller($\alpha_i(t+1) = 0$) in the next moment. Let in this case he can hope to buy the commodity by the price which is less than the price which he asked in moment t (v_i(t)>u(t)).

If in this case he has money ($y_i(t+1) > 0$) then he becomes the buyer ($\alpha_i(t+1=-1$), if has no money($yit+1=0$) he must wait the appropriate situation and becomes the waiting agent($\alpha_i(t+1) = 0$). If ($v_i(t) \leq u(t)$) then there has sense for seller i($\alpha_i(t) = 1$) to remain the seller($\alpha_i(t+1) = 1$).

Now we shall consider case when our agent in moment t is a buyer (α_i(t)=-1).

If in the first tact of moment t the buyer has spent all his money which he has had ($y_i(t+1) = 0$) then in moment t+1 he can became seller ($\alpha_i(t+1) = 0$)or waiting agent($\alpha_i(t+1) = 0$) only. It is profitable for him to become the seller if he can hope to sell the commodity in next moment (t+1) more expensively than he has bought the commodity in moment t ($w_i(t) < u(t)$). We can propose that he make the decision in accordance to the following algorithm. If $w_i(t) < u(t)$he will become the seller($\alpha_i(t+1)=1$). In other case (w_i(t)≥ $u(t)$) he will become waiting agent($\alpha_i(t+1)=0$) .

If in the first tact of moment t the buyer spent only part of his money or not took part in trade ($y_i(t+1)>0$)then there is one reason of it. This reason is his price($v_i(t)$) which is rather low for all sellers in this moment(t). If he has commodity and has money ($x_i(t+1)\geq 0, y_i(t+1)>0$)then he can choose to become a seller($\alpha_i(t+1)=1$) or a buyer($\alpha_i(t+1)=-1$)in the next moment(t+1). If $v_i(t)<u(t)$ then he can hope to sell the commodity in the moment t +1 by the price which is more than the price which he asked in moment t ($v_i(t)$). If in this case he has commodity ($x_i(t+1)>0$) then he becomes the seller($\alpha_i(t+1)=1$) . In the opposite case($x_i(t+1)=0$) he must wait the appropriate situation and becomes the waiting agent ($\alpha_i(t+1)=0$). If $v_i(t)\geq u(t)$ then there has sense to remain the buyer ($\alpha_i(t+1)=1$).

Finally we shall consider the case when the our agent is a waiting agent in the moment t($\alpha_i(t)=0$). For such agent the variables which determine his decision are the quantities of commodity and money which he has his price, his relation to risk in this moment and average price of market in moment t ($x_i(t), y_i(t), v_i(t), k_i(t), u(t)$). If $v_i(t)>u(t)$ b then has sense for this agent to become the buyer ($\alpha_i(t+1)=-1$) . If he has money for this ($y_i(t)=y_i(t+1)>0$) a he become the buyer. If he has no money then he remain the waiting agent in next moment of time ($\alpha_i(t+1)=0$).). If $v_i(t)<u(t)$ then has sense for this agent to become the seller. If he has commodity for this ($x_i(t)=x_i(t+1)>0$) he become the seller($\alpha_i(t+1)=1$). If he has no commodity then he remain the waiting agent ($\alpha_i(t+1)=0$).)in next moment of time (t+1).

The definition of new statuses of participant is not the end of the second tact of moment of time t. Participants after it establish their new prices in next moment of time(v_i(t+1))). We shall suppose (to simplify the model) that participant can decrease his price or can increase his price on the value d only . d is the same in all moments of time and it is a parameter of model. Now we shall describe simple rules of changing of prices related to following two kinds of participants: a careful agent in moment t (k_i(t+1)=1) and a risky agent in this moment of time ($k_i(t+1) = -1$). The difference between careful and risky agent consists in the following. When agent wants to sell then if this agent is risky agent then he tries to ask the price which is more on value d of the price which careful agent should define in this case (if it possible). When agent wants to buy then if this agent is risky agent then he tries to propose the price which is less on the value d of the price which careful agent should define in this case . We shall describe the algorithm of changing of prices below.

If a seller have sold all his commodity($x_i(t+1) = 0$) or a buyer have spent all his money (y_i(t+1)=0)in the first tact of the moment t then he can use average price of all his bargains($w_i(t)$)) at the definition of a price in next moment (t+1). This average price ($w_i(t)$)of agent is more of his price in moment t ($v_i(t)$), if he was s seller and ($w_i(t)$) it is less of his price in moment t ($v_i(t)$) if he was buyer. If in this situation the participant was a seller ($\alpha_i(t) = 1$)) in moment t and became a buyer(α_i(t+1)=-1) in moment t+1, then the careful agent t chooses the new price which equal his average price in moment r ($v_i(t+1) = w_i(t)$) but a risky agent chooses the price which is less of average price of agent in this moment on value d ($v_i(t+1) = w_i(t)) - d$), if it possible. If it not possible he does as careful agent ($v_i(t+1) = w_i(t)$)). If in this situation the participant was a buyer (α_i(t)=-1) in moment t and became a seller (α_i(t+1)=1) in moment t+1 then he do the same ($v_i(t+1) = w_i(t)$)) but in case of risky agent he defines the price which more of average price of agent on value d($v_i(t+1) = w_i(t)) + d$), if it possible.

Let consider once more the situation in which a seller has sold all his commodity or in this situation a buyer has spent all his money in the first tact of the moment t. If some participant was a seller (α_i(t)=1) in moment t and became a waiting agent in moment t+1(α_i(t+1)=0), then the careful agent t chooses the new price which equal his average price in moment r (($v_i(t+1) = w_i(t)$))) but a risky agent chooses the price which is less of his average price in this moment on value d $((v_i(t+1) = w_i(t)) - d)$). If in this situation the participant was a buyer (α_i(t)=1) in moment t and became a waiting agent (α_i(t+1)=0)in moment t+1 then he do the same but in case of risky agent he defines the price which more of average price on value d $(v_i(t+1) = w_i(t)) + d)$).

Now we shall consider the remaining possibility. This will be when seller have sold in the first tact of moment t only part of his commodity or he at all not took part in exchange ($x_i(t+1) > 0$) or when buyer have spent in the first tact of moment t only part of his money or he at all not took part in exchange($y_i(t+1>0)$. In this case participant can be guided by his price in moment t (v_i(t))and by the result of his exchange($x_i(t+1), y_i$(t+1))or absence it in first tact of moment t. If in this case the agent which was a seller in moment t ($\alpha_i(t) = 1$) becomes the buyer in moment t+1 ($\alpha_i(t+1) = -1$) then a careful agent chooses his price in moment t as his price in moment t+1($v_i(t+1) = v_i(t)$). The risky agent chooses the price in moment t+1 the price which is less his price in moment t on value d($v_i(t+1) = v_i(t) - d$), if it possible ($v_i(t) - d \geq u(t)$) and he chooses his previous price ($v_i(t)$) if it is impossible($v_i(t+1) = v_i(t)$). The agent which was a buyer in moment t ($\alpha_i(t) = -1$)and becomes a seller in moment t+1 ($\alpha_i(t+1) = 1$)do the same with one difference that risky agent increases his price in moment t+1,if it possible ($v_i(t) + d \leq u(t)$). If in this case the agent which was a seller in moment t ($\alpha_i(t) = 1$) becomes the waiting agent in moment t+1 ($\alpha_i(t) = 0$) then a careful agent chooses his price in moment t as his price in moment t+1($v_i(t+1) = v_i(t)$). The risky agent chooses the price in moment t+1 the price which is more his price in moment t on value d($(v_i(t+1) = v_i(t) + d)$). The agent which was a buyer in moment t ($\alpha_i(t) = -1$))and becomes a waiting agent in moment t+1 ($\alpha_i(t+1) = 0$) do the same with one difference that risky agent decreases his price in moment t+1($v_i(t+1) = v_i(t) - d)$),if it possible ($v_i(t) - d \geq u(t)$)). If in this case the agent which was a seller in moment t ($\alpha_i(t) = 1$)remains the seller in moment t+1 ($\alpha_i(t+1) = 1$) then a careful agent chooses his price in moment t+1 which is less his price in moment t on value d($(t+1) = v_i(t) - d$). The risky agent chooses the price in moment t+1 the price which more his price in moment t on value d($v_i(t+1) = v_i(t) + d)$, if it possible (($v_i(t) + d \leq u(t)$) and he chooses his previous price ($v_i(t)$) if it is impossible(($v_i(t) - d > u(t)$).

The agent which was a buyer in moment t ($\alpha_i(t) = -1$) and remains a buyer ($\alpha_i(t+1) = -1$)in moment t+1 do the same with one difference that careful agent increases his price($v_i(t+1) = v_i(t) + d$))and risky agent decreases his price in moment t+1 ($v_i(t+1) = v_i(t) - d$)),if it possible ($v_i(t) - d \geq u(t)$)).

If the agent is a waiting agent in moment t ($\alpha_i(t) = 0$) then his quantity of commodity and quantity of money are the same in moment t+1 ($x_i(t+1) = x_i(t), y_i(t+1) = y_i * t$)). If a waiting agent($\alpha_i(t) = 0$) t becomes the seller ($\alpha_i(t+1) = 1$) t then he establish new price by the following algorithm. When he is careful agent his price in moment t+1 is equal his price in moment t ($v_i(t+1=vi(t)$). If he is risky agent then his price in moment t+1 is more his price in moment t on value d if it is possible ($v_i(t+1) = v_i(t) + d$), if it is impossible ($v_i(t) + d \leq u(t)$) then both prices are equal.

If a waiting agent becomes the buyer then he establish new price by the following algorithm. When he is careful agent his price in moment t+1 is equal his price in moment t ($v_i(t+1) = v_i(t)$). If he is risky agent then his price in moment t+1 is less his price in moment t on value d ($v_i(t+1) = v_i(t) - d$) if it is possible ($v_i(t) - d \leq u(t)$)), if it is impossible then both prices are equal

.If he is a waiting agent remain the waiting agent then he must take in account the variation of average price of market) i.e u(t)-u(t-1) .

If he has a commodity and he is a careful agent then he decrease his price on value d ($v_i(t+1) = v_i(t) - d$)). If he has commodity and he is risky agent then decrease his price($v_i(t+1) = v_i(t) - d$) on value d if u(t)-u(t-1) nonpositive and not change his price in the opposite case.

If he has no commodity but he has some money and he is a careful agent then he increase his price on value d ($v_i(t+1) = v_i(t) + d$)). If he has no commodity but he has some money and he is risky agent then increase his price on value d$v_i(t+1) = v_i(t) + d$) if u(t)-u(t-1) nonnegative and not change his price i$v_i(t+1) = v_i(t)$)n the opposite case.

So we have described the changing of prices of participants of our model of market. It is important to note that our algorithm of change of statuses and prices by participants is not the alone which is logically justified one. We can write this algorithm by formulas. We shall use the Heavisid function:

$$\theta(x) = \begin{cases} o\ if\ x < 0 \\ 1\ if\ \ x \geq 0 \end{cases}$$

The definition of the status of a participant i in moment t+1 is a following.

if $\alpha_i(t) = 1$ then

when $x_i(t+1) = 0$ $\alpha_i(t+1) = -[(1 - \theta\big(u(t) - w_i(t)\big)]$

when $x_i(t+1) > 0$ $\alpha_i(t+1) = 1 - \theta(v_i(t) - u(t)(1 + \theta(-y_i(t+1)))$

if $\alpha_i(t) = -1$ then

when $y_i(t+1) = 0$ $\alpha_i(t+1) = 1 - \theta(w_i(t) - u(t))$

when $y_i(t+1) > 0$ $\alpha_i(t+1) = -1 + \theta(u(t) - v_i(t)(1 + \theta(-x_i(t+1))$

if $\alpha_i(t) = 0$ then

when $y_i(t+1) = 0$ $\alpha_i(t+1) = \theta(u(t) - v_i(t))$

when $y_i(t+1) > 0$ $\alpha_i(t+1) = -\theta(v_i(t) - u(t))$

The condition $min_{\alpha_i(t+1)=-1} v_i(t+1) \geq max_{\alpha_i(t+1)=1} v_i(t)$ requires:

If $\alpha_i(t+1) = 1$ then must be $v_i(t+1) \leq u(t)$

If $\alpha_i(t+1) = -1$ then must be $v_i(t+1) \geq u(t)$

If $\alpha_i(t+1) = 0$ and $y_i(t+1) = 0$ then must be $v_i(t+1) \geq u(t)$

If $\alpha_i(t+1) = 0$ and $y_i\ (t+1) > 0$ then must be $v_i(t+1) \leq u(t)$

If a risky agent wants to define his price which is not satisfying to these conditions then he cannot do it.

The definition of the price of a participant i in moment t+1 (which fulfill to these conditions)is a following.

If α_i(t)=1 and x_i(t+1)=0 then v_i(t+1)=$w_i(t)$ when $k_i(t) = 1$ but $v_i(t+1) = w_i$(t)-d$\theta(w_i(t) - d - u(t))$ when $k_i(t) = -1$.

If α_i(t)=-1 and y_i(t+1)=0 then v_i(t+1)=$w_i(t)$ when $k_i(t) = 1$ but $v_i(t+ 1=wi$(t)+d$\theta ut-wit-d$ when kit+-1

If α_i(t)=1 and x_i(t+1)>0 then v_i(t+1)=$v_i(t) - d[1 - \theta(u(t) - v_i(t))]$ $when\ k_i(t) = 1$ but $v_i(t+1) = v_i$(t)+d{$(1-\theta)\theta(u(t) - v_i$(t)-d)+ $\theta(v_i$(t)-u(t))[$\theta(-y_i$(t+1))$\theta(v_i(t) - u(t) - d) + 1 - \theta(-y_i$(t+1))]} when $k_i(t) = -1$.

If α_i(t)=-1 and y_i(t+1)>0 then v_i(t+1)=$v_i(t) + d\{1 - \theta(v_i(t) - u(t)]$ $when\ k_i(t) = 1$ but $v_i(t+1) = v_i(t) + d\{(1 - \theta(u(t) - v_i$(t)))$\theta(v_i$(t)-u(t)-d)+$\theta\big(u(t) - v_i(t)\big)[\theta(-x_i(t+1))\ \theta(v_i$(t)-u(t)-d)+1-$\theta(-x_i$(t+1))]} when $k_i(t) = -1$.

If α_i(t)=0 and x_i(t+1)>0 then v_i(t+1)=$v_i(t) - d[1 - \theta\big(u(t) - v_i(t)\big)]$ $when\ k_i(t) = 1$ but $v_i(t+1) = v_i$(t)+d{$\theta\big(u(t) - v_i(t)\big) - (1 - \theta\big(u(t) - v_i(t)\big))\theta(u(t-1) - u(t)\}$ when $k_i(t) = -1$.

If α_i(t)=0 and x_i(t+1)=0 then v_i(t+1)=$v_i(t) + d[1 - \theta(v_i(t) - u(t))]$ but $v_i(t+1) = v_i(t) - d[\theta\big(u(t) - v_i(t)\big)\theta(v_i(t) - u(t) - d) - (1 - \theta\big(u(t) - vit\theta ut-ut-1]$

These formulas give to us possibility to find r(t+1) as a function of r(t) ,u(t)and u(t-1).

3. The characteristics of the spectrum of prices

We shall try to give the description of the set of variables $v_i(t)(i = 1,2,3,..,N)$. We shall name this set by spectrum of prices just the same way it was in our previous works.

Denote by ρ(t)=$max_{\alpha_i(t)=1} v_i(t) - min_{\alpha_i(t)=-1} v_i(t)$ and we shall name this value by width of spectrum of prices of participants of trade in moment of time t..

Denote also σ(t)=$min_{\alpha_i(t)=-1} - max_{\alpha_i t)=1} v_i(t)$ and we shall name this value by divergence of spectrum of prices of participants of trade in moment of time t

We can see the following inequalities as a consequences from the algorithm of change of statuses and prices which have been described in the previous section . We remark that according the our algorithm for participant (careful and risky) which became the seller in moment t+1 must be $v_i(t+1) \le u(t)$) and for participant (careful and risky) which became the buyer in moment t+1 must be $v_i(t+1) \ge u(t)$). Also we can note that participant of market becomes in moment t+1 a waiting agent when he was seller or buyer in moment t if $x_i(T + 1=0$ or if $yit+1=0$ only. Except it we can see that for any waiting agent exist some moment when he become seller or buyer. We have assertion 1.

Assertion 1. The exist some moment of time T_0 that begginning with moment of time T_0 wil be ρ(t)>0, σ(t) >0 for t> T_0.If $\alpha_i(t) = 0$ then for t>T_o will be either x(t)=0 or y(t)=0.

Let us to introduce the following designations:

$I_1(t)$) -the set of all indexes of participants (i) which are sellers in moment t($\alpha_i(t) = 1$,

$I_{-1}(t)$-the set of all indexes of participants (i) which are sellers in moment t($\alpha_i(t) = -1$),

I_0^x(t)-the set of all indexes of participants (i) which are waiting agents in moment t and which have some quantity of the commodity ($\alpha_i(t) = 0, x_i(t) > 0$),

$I_0^y(t)$-the set of all indexes of participants which are waiting agents in moment t and which have no any quantity of the commodity ($\alpha_0(t) = 0, x_i(t) = 0$)

$\eta_1(t) = u\ (t-1) - min_{\alpha_{i(t)=1}} v_i(t), \eta_{-1}(t) = max_{\alpha_{i(t)=-1}} v_i(t) - u(t-!)$,

$$\nu_x(t) = max_{\alpha_{i(}(t)=0, x_i(t)>0} v_i(t) - u(t-1)$$

$$\xi_x(t) = u(t-1) - min_{\alpha_i(t)=0, x_i(t)>0} v_i(t)$$

$$\nu_y(t) = max_{\alpha_{i(}(t)=0, y_i(t)>0} v_i(t) - u(t-1)$$

$$\xi_y(t) = u(t-1) - min_{\alpha_i(t)=0, y_i(t)>0} v_i(t)$$

The assertion 2 , 3 an 4 follow from the our algorithm also

Assertion 2. For t>$T_0 + 1$ *we have* $\eta_1(t) \geq 0, \eta_{-1}(t) \geq 0, d \geq \xi_x(t) \geq 0, d \geq \nu_y(t) \geq 0, \xi_y(t) \geq 0, \nu_x(t) \geq 0$.

Assertion 3. The such T_1 *exist that for any t>* T_1*will be:* $\rho(t) \leq 5d + \beta, \eta_1(t) \leq 3.5d + \frac{\beta}{2}, \eta_{-1}(t) \leq 3.5d + \frac{\beta}{2}$, *where* β *is as much as desired small but constant value.*

Assertion 4.If u(t-1) is sufficiently large then in the case when u(t)-u(t-1)>0 will prove to be u(t+1)-u(t)<0 or u(t+2)-u(t+1)<0. If u(t-1) is sufficiently small then in the case when u(t)-u (t-1)<0 will p rove to be u(t+1)-u(t)>0 or u(t+2)-u(t+1)>0.

The consequence 1 from assertion 4.The maximal time in which the participant i not change his status is less or equal four moment of time.

The consequence2 from assertions 3,4.There exist such $T_2 > T_1$ *that for any t>*T_2 *will be* $\xi_y(t) \leq 3.5d + \beta, \nu_x(t) \leq 3.5d + \beta$.

(The proofs of assertion 1-4 and the consequences from assertion 4 see in [14]

4. The investigation of the model with the identical and constant risk relation by computer experiments.

The set of 5N numbers $x_1(t)y_1(t), v_1(t), \alpha_1(t), k_1(t), x_2(t)y_2(t)v_2(t)\alpha_2(t)$, $k_2(t), \dots, x_N(t), y_i(t), v_i(t), \alpha_N(t), k_n(t)$ defines the state of our system in moment of time t. We shall denote this set of numbers (state of our system) by r(t). Let to note that the value $\alpha_i(T)$) takes one from three meaning 1,0,-1 and it is changing during the time and value $k_i(t)$ takes one from two meanings and in our case it is constant during the time. Values $x_i(t)$ and $y_i(t)$ take meaning in interval [0,1] with conditions: $\sum_{i=1}^{i=N} x_i(t) = 1$, $\sum_{i=1}^{i=N} y_i(t) = 1$ .Variable $v_i(t)$ takes meaning in the some interval . The algorithm of transformation from r(t) to r(y+1) was described in the section 2, i.e. dynamics of the system have been defined. The state of system for which r(t+1)=r(t) is named by steady state.

Model of closed market which was described above was found very complicated for analytical investigation. At least author could not do it with success. It is the first reason of that we had created the computer model of our system by using strict logical description of model from second section of this paper. We can mention simple example of the steady state. Let we have system which consists from one buyer with .5 units of money, one seller which has .5 units of commodity, one waiting agent with .5 units of money and one waiting agent with .5 units of commodity and let each agent has prices which equals units. The price of bargain between buyer and seller will be unit and accordingly with our algorithm we shall have in next moment the same state of system that have been in the initial moment. We could to mention also simple examples some cyclic trajectories when r(t+2)=r(t) , but there was no steady states and cyclic trajectories in our computer experiments with this system which has many participants(N>100). But we can see in the experiments that our trajectory belongs to some set of states of system all time after some moment of time. We shall name such set of states of system MM_0 for which if r(t)∈ M_0M then should be r(t+1)∈ M_0<M by stationary set of states. Let denote :

$X_i(t) = \sum_{\alpha_i(t)=1} x_i(t)$,$Y_i(t) = \sum_{\alpha_i(t)=1} y_{i(t)}$

$X_{-1}(t) = \sum_{\alpha_i(t)=-1} x_i\,(t)$,$Y_{-i}(t) = \sum_{\alpha_i(t)=-1} y_{i(t)}$

Let note the following. $X_1(t)$X is the common quantity of commodity , which sellers are proposing for sale and $Y_{-1}(t)$Y is the common quantity of money which is destined for buy of commodity on the first step of moment of time t.

The choice of the initial state of system has a great importance for investigation at computer investigation of system. It is obviously that there are some initial states at which system very quickly will be in corresponding stationary set. For example the such initial condition is a state system in which a part of participants has no money and commodity but other part of participants has all money and commodity. We choose as a initial state a state r(0) when the distributions of unit of commodity and unit money between participant is close to uniform distribution. We have chosen $x_i(0)$ and $y_i(t)$ distributed uniformly on the interval between zero and one. After it we normalised it in order the sum of commodity become equal unit and sum of money in system become equal unit.

We had investigated our model of closed market in many computer experiments by observations of trajectory of some general variables of system during a very long time of development of our system (more than 20000 moments of time) . First of all the result of all experiments is the following correction of analytical results of third section. There exist such τ_1 that for any t> $\tau_1$there is the following:

$$0 \le \rho(t) \le 4d, \sigma(t)) \ge 0$$

$$0 \le \eta_1(t) \le 2.5d, 0 \le \eta_{-1}(t) \le 2.5d$$

$$0 \le \xi_x(t) \le d, 0 \le \xi_y(t) \le 3.5d$$

$$0 \le \nu_x(t) \le 1.25d, 0 \le \nu_y(t) \le 1.25d$$

These experimental results concern the case when $k_i(t)(i = 1,2,$ ...,N,0$\le t \le \infty)$ are constant and they are equal during the experiment 1 or -1. But we investigated separately in the first part of our experiments the case when $k_i(t) = 1(i =$

$1,2,\dots,N)$ not change during the time and the case when $k_i(t) = -1(i = 1,2,\dots,N)$ not change during the time .

At first we investigated the first case ,

The following experimental results concerns to first case only. We consider the case at first when all participants of model are careful in all moments of time . The two our papers [14],[15] was devoted to investigation of dynamics of model in this same case. Now we also begin the investigation of our model in case when k_i(t)=1 (i=1,2,…,N) for 0≤ t ≤ ∞.

The convergence of trajectory of system to some stationary set if the most important conclusion from our computer investigation of dynamics of the system.

If $k_i(t) = 1(i = 1,2,\dots,N)$ *during all time then there exists such moment of time* τ *that for each t>* τ *takes place r(t)∈* $M_0(d)$*, where for all points of set* $M_0(d)M$ *will be* $u_o - 5d \leq u(t) \leq u_o$*+5d.*

Unfortunately it is unique property of set M_o which we was able to establish until now. But it is may most important result of our investigation. Analytical proof of this fact and other our results is connected with very large difficulties which we cannot overcome.

We can see on the figure 1 the change of average price of market for two experiments: the first when trajectory begins from the large value of average price and the second when trajectory begins from small value of average price of market. In both cases after some time τ both trajectories belong to $M_0(d)$.

Our more accurate observation of computer trajectory of average price of the market have shown to us that trajectory tends to set $M_0(d)$ not monotonically and her changes in this set is not monotonic also but it hesitates near his average value. There are two kinds of hesitations of u(t) : first kind is a hesitations in short time (a increasing of u(t) after a decreasing of u(t) during several moments) and not regular hesitations of u(t) during of large time (with large and not constant period)

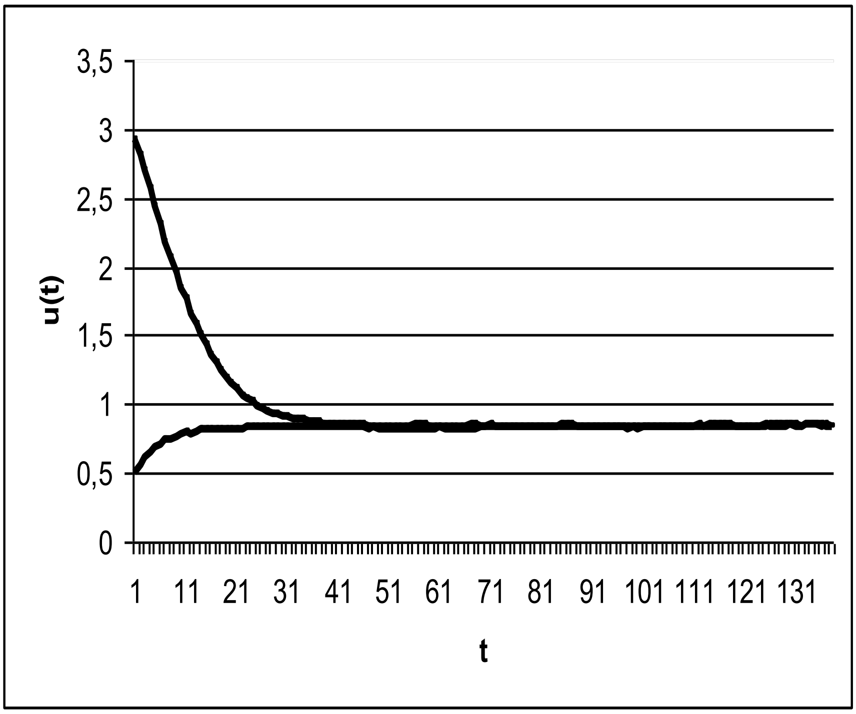

Figure 1 The graph of change of average prices of closed market during the time for two initial conditions (N=300,d=0.005,u(0)= 3 u(0)=0.5, one step in axis of time on the graph equals 100 moment of time).

We can see from graph 2 that these hesitations are not regular also when $r(t) \in M_o(d)$ and periods of these hesitations are not equal (we have not observed any regularity of these hesitations).

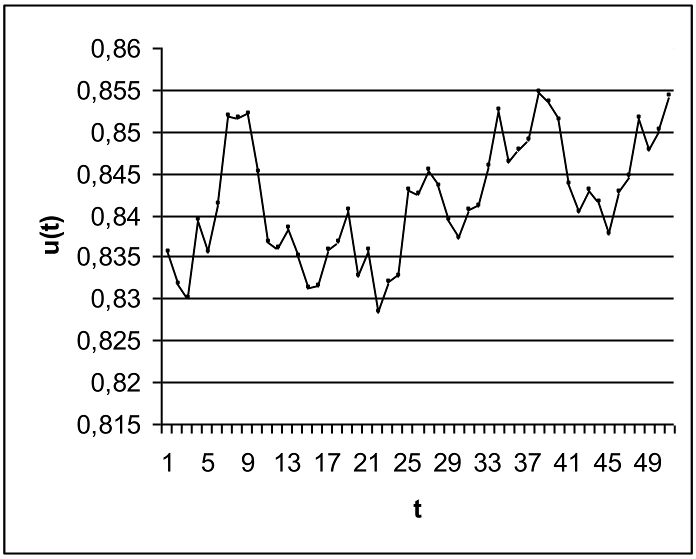

Figure 2 The graph of change of average prices of closed market u(t) during the time when r(t)$M_0(d)$. (N=300,d=0.005,u(0)= 3 , one step in axis of time on the graph equals 100 moment of time 8000$\leq t \leq 13000$).

There are the initial value of u(t) is 3 , and we consider trajectory u(t) in interval time from 8000 moments until 13000 moments, when r(t) $\in M_o(d)$. The average u(t) on this interval is 0.8378($u_1 = 0.8378$), the minimal value of u(t) is 0.8271 and the maximal value of u(t) on this interval is 0.8488. So we have on this interval $u_1 - 2.1d < u(t) < u_1 + 2.2d$.

If r(t)$\in M_0(d)$, then it is not necessary that r(t) is equilibrium state. The supply which is evaluated in money can be not equal to a demand (quantity of money to purchases) in most of moments of time. In most computer experiments when r(t)$\in M_0(d)$ the value $|X_1(t)u(t0 - Y_{-1}(t)|$ is rather large. Really, there are some number of waiting agents which have money and commodity, which take not part in trade in this moment. It is interesting to consider a periods which consist from several sequential moment of time. Accordingly consequence 1 from assertion 4 the each agent can be continuously in one from a varying sets of agents $(I_1(t), I_{-1}(t) I_0^x(t) I_0^y$(t) not more than 4 moments of time. Is not difficult to understand from this remark that each participant changes several statuses and after the some time return to the initial status. These arguments can partly explain the one experimental fact. Let denote by $D_L(t)$ the average value of absolute value of averaged for several (L) previous moments of time the difference between the demand which is evaluated in money and quantity of money which all buyers have :

$$D_l(t) = |\sum_{s=t-L+1}^{s=t} \big(X_1(s)u(s) - Y_{-1}(t)\big)|/L$$

Our computer experiments have shown for 17<L<25 it had turned out be $D_l(t) < 30d$ if r(t)$\in M_0(d)$. So we can understand it as a equilibrium on the average. In particular the figure 3 say to us that states from set $M_o(d)$ correspond to equilibrium on the average for l8 previous moments of time (at L=18).

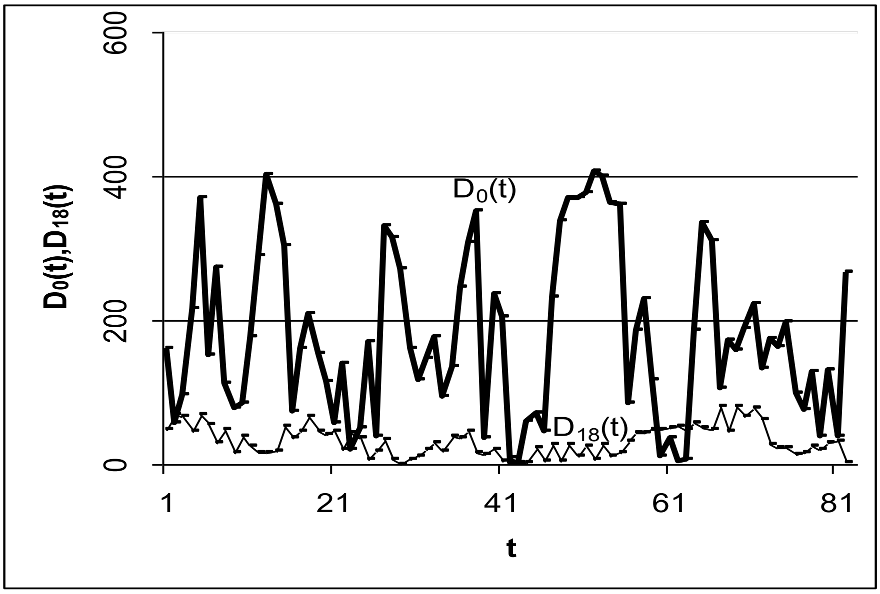

Figure 3.The difference between the effective demand and supply and the same difference which was averaged during the 20 previous moments of time for the trajectory entirely belonging to the set $M_0(d)$ (n=300,d=0.002 9500<t<9600)

We shall name the value $x_i(t)u(t) + y_i(t)$ by asset of participant with index i in moment of time t. It is obviously that average asset of participant of market equals $\frac{(u(t)+1)}{N}$. We investigated of the dynamics of the ratio of square root of the sum of the deviates of asset each agent from the average asset to the average value of asset of agent in moment t . We denotes this value by S(t).

$$S(t) = \{\sum_{i=1}^{i=N} [x_i(t)u(t) + y_i(t) - (1 + u(t))/N]^2\}^{1/2}/(1 + u(t))$$

We can see the very slow reduction S(t) with the during the time in fig.4
And we see the reason of it in the fact that all participants have the same mechanism of decision making –they are all careful.

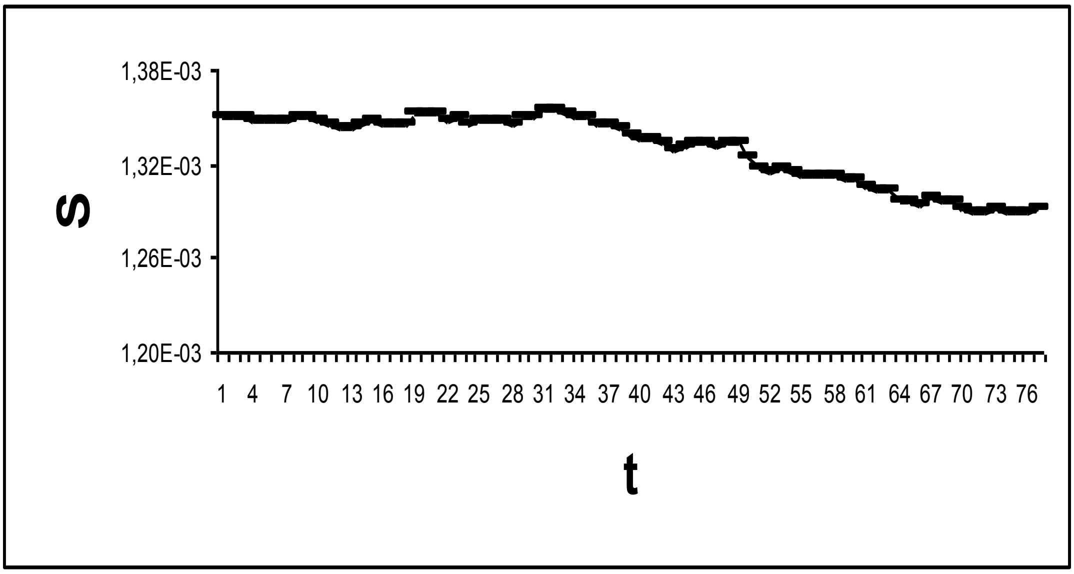

Figure 4. The dynamics of S(t) for case when d=0.005,N=500, 0<t<15000

Now we can discuss the behavior of system when there are risky agents as participants of our market. At first we consider the case when all participants are risky agents. In such case we cannot say about convergence of trajectory of system to some stationary set of states but some quasiregular regime of hesitations is establishing for trajectory of the average price of the market . All trajectories of u(t) was situated inside of the rather large interval $u_1 - 200d < u(t) < u_1\_200d$ when $u_2 = 1$ and 4000<t< 1000000 in very large quantity of experiments . That was for any case of initial conditions which are corresponding to the description of section 2. It took place at all initial values of u(t) from u(0)=0.2 until u(0)=20. But for t>2000 in all our experiments u(t) hesitated. But these hesitations was not regular. There were no unique period or one amplitude of hesitation. Periods of hesitation were different, but it was not less the 1500 moment of time and not more 2300 moments of time. The values of u(t) hesitates also was not less 100d and not more 200d. We can see the examples of this situation on the fig. 5 where we have two trajectories of u(t): one which begins from u(0)=0.2< second which begins from u(0)=3.We calculated average values, minimal and maximal values for interval 2000<t<14000. For the one graphic (for which u(0)=0.2) the average value u(t) equals 0.8788, minimal value 0.1764 and maximal value equals 1.7182. For the second graphic (for which u(0)=0.2) the average value u(t) equals 1.0446, minimal value 0.5670 and maximal value equals 1.8414

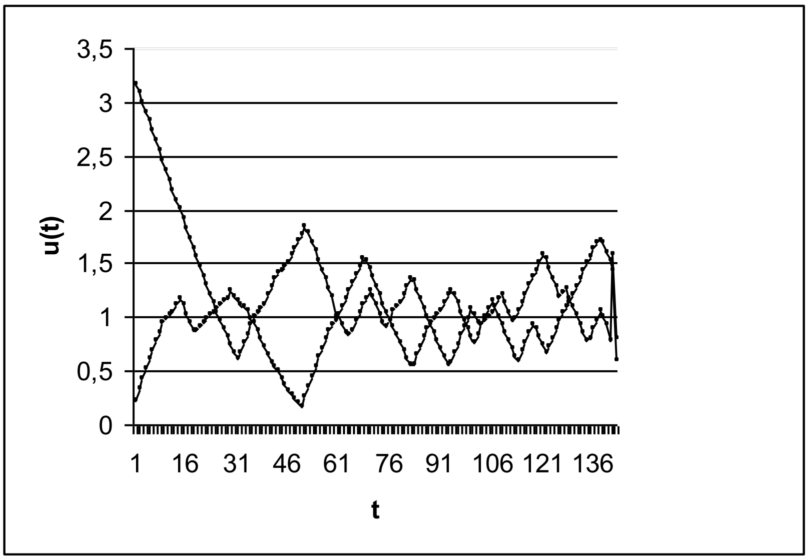

Figure 5. Two trajectories of average price of market when all participants are risky agents.(N=500, d=0.005,u(0)=0.2 and u(0)=3.3. 0<t<14000).

It is necessary to note that in small diapason of time (about 100 moments of time) trajectory has small hesitation i.e. increase after decrease . We could see similar picture above in the case when all participants are careful. Dynamics of S(t) is very similar to the dynamics in the case when all agents are careful i.e. S(t) changes in very small way on the large period of time. There are two reason of this phenomenon : the first is that in initial condition a assets are distributed uniformly among participants, the second is that mechanism of choice of status and price is the same for all participants.

It is very interesting to consider the case when the part of participants of market are careful agents and other participants are risky agents. There are many variants of this situation but we consider in this paper only one. In the initial moment of time each participant prefers to be careful agent in all time of experiment with probability 0.5, consequently he is risky agent in all moments of time with probability 0.5 also. First of all the trajectory of u(t) is interesting for us.

Many computer trajectories with different initial conditions (which are corresponding to section 2) show us fundamental properties of behavior of u(t) during the long time . u(t) converges to some interval of its values and after some moment of time it is in this interval of values. If u2 is a average value of u(t) after moment of first location in this interval until time of finish of the experiment then during the his time will be; $u2-50d<u(t)<u2+50d$. This interval is much more than the same interval in the case all careful agents and less than same interval in the case of all risky agents. We can see in the fig 6 the graph of u(t) where u2=1,0344 and after t=4000 we have $u2-10d<u(t)<u2+50d$.

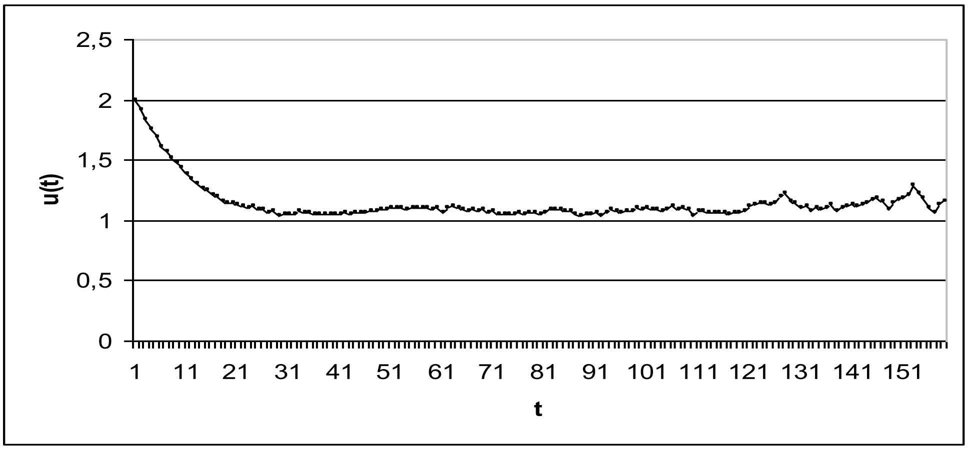

Figure 6. The trajectory of average price of the market in the case where about half of agents are careful and other agents are risky agents. .(N=500, d=0.005,u(0)=2. $0<t<15500$).

The behavior of value S(t) in tis case is rather interesting. Tbe S(t) increases with increasing of time. It say to us that risky choice tends to growth of assets of participant. We can see in fig. 7 that during the 15200 moments of time the S(t) increases from .02 until 0.08. For more clearness we can mention the following dates. In The initial moment of time of one from our experiments the summary quantity assets of all careful agents was 1.512 the same for all risky agents was 1.488 at initial average price of market equal 2. After 15200 moments of time the summary quantity assets of all careful agents was 0.1698 the same for all risky agents was 1.9302 at average price of market equal 1.1. in the same experiment.

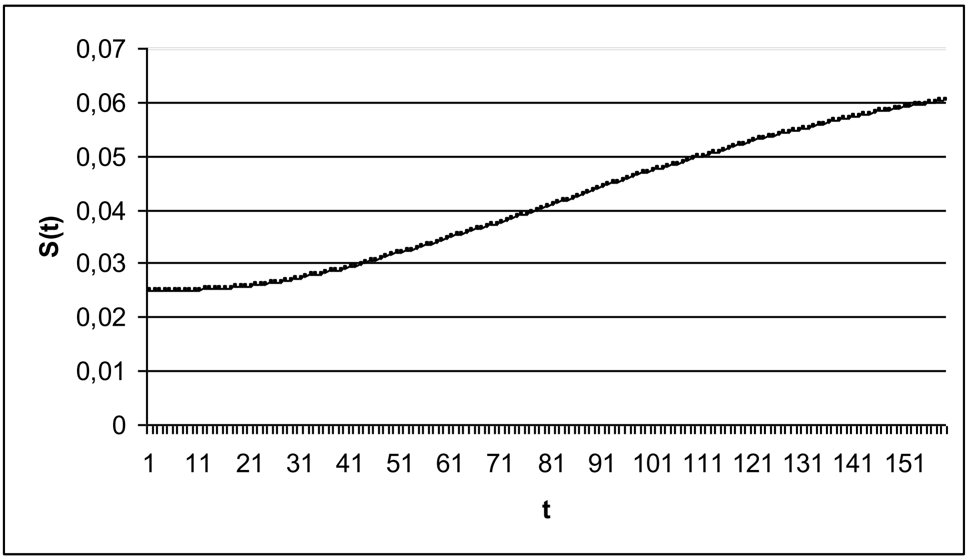

Figure 7. the graph of S(t) in the case where about half of agents are careful and other agents are risky agents. .(N=500, d=0.005,u(0)=2. 0<t<15500).

5 The dynamic game of automata as model of closed market with single commodity

Participants of market choose the status in the moment of time t+1 by using comparison of average price of all their bargains in the moment t with the average price of all bargains of the market i.e. average price of market in this moment of time. We think that this mechanism of choice is necessary for regularity of spectrum of prices , it is logically justified and rather simple. But each participant was careful agent or risky agent when he chose the price for next moment during the all time . We shall consider in this section the market where each participant can choose to be careful agent or risky agent in this moment as a consequence of his successful result or unsuccessful result in previous moment.

We shall use in this section the results of I.M Gelfant and M.L. Tsetlin about behavior of finite automata and modeling of simplest forms of behavior (see [1],[2]) in particular we shall use their model of behavior of automata in stationary random media-We must remind of some definitions and some results from their theory.

There are n actions of our deterministic automata which we denote by $k_1, k_2, k_3, \dots, k_n$. If k(t)-fi then we shall say that automaton fulfils the action f_i in moment t. There are M states of automaton which we shall denote by $\varphi_1, \varphi_2, \varphi_3, \dots, \varphi_M$ (M$\geq n$) and there are some correspondence between states and actions of automaton. M is named memory capacity. Where t represents time and it assumed to take integer value 1,2,...,. There is some input variable s(t) which depends on time and which can take only two value (1,0) The value s=0 is called the "nonpenalty and the value s=1 is called the "penalty" of an automaton. The dynamics of deterministic automaton is described by two equalities:

$$\varphi(t+1) = \Phi\big(\varphi(t), s(t+1)\big), k(t) = F(\varphi(t))$$

The matrix ‖ $a_{i,j}(s)$ ‖ determines the transitions of states for a deterministic automaton in the following manner: if at the instant t the automaton is in state φ_i , then at t + 1 it will make a transition to a state φ_j such that $a_{i,j}(s(t+1)) = 1$., for l≠j we have $a_{i,l}(s(t+1)) = 0$.,

An automaton is in a stationary random medium $C = C(a_1, a_2, a3 \ , \dots, a_K)$ if the actions of the automaton and the values of its input variable are related as follows: the action k_ω(ωs= 1,2,3, ...,n), if, it performed by the automaton at the moment t / generates the value s =1 (a penalty) in the moment t +1 with the probability $p_\omega = \frac{1-a_\omega}{2}$ and the value s = 0 (a nonpenalty) with the probability q_ω=$\frac{1+a_\omega}{2}$. We assume here that $|a_\omega| \ll 1$.

The probabilities $p_{i,j}$of the transition of the automaton from state φ_i (to which corresponds the action $f_{\omega_i}, f_{\omega_i} = F(\varphi_i)$) to state φ_j is given by the formula: $p_{1,j}$=$p_{\omega_i} a_{i,j}(1) + q_{\omega_i} a_{i,j}(o)$ where $a_{i,j}(1) = 1$ if $\varphi(t+1) = \varphi_j$ when

$\varphi(t) = \varphi_i$ and s(t)=1 and $a_{i,j}(1) = 0$ if $\varphi(t+1) \neq \varphi_j$ when $\varphi(t) = \varphi_i$ and s(t)=1/($a_{i,j}(0)$ means the same when s(t)=0) .Also the matrix $P = \| p_{i,j} \|$ is stochastic.

Therefore the functioning of the automaton in a stationary random medium is described by a Markov chain. When this chain turns out to be ergodic so that final probabilities of automaton states exist in a given medium which are independent of its initial state. This situation is in the most cases which were investigated in this theory. An automaton is named symmetric if the expected value of a nonpenalty in any stationary random medium is a symmetric function of $a_2, a_2, a_3, \dots, a_k$.l Denote W©) the expected notpenalty for the automaton in the media C/

A sequence of automata. . , an ,. . . will be called asymptotically optimal if $\lim_{m\to\infty} W(C) = max(a_1, a_2, a_3, \dots, a_K)$.

An automaton belonging to an asymptotically optimal sequence, if m is sufficiently large, performs almost exclusively the action for which the probability of a nonpenalty is maximum.

We shall use in our model one simplest asymptotically optimal automaton which was proposed and investigated by M.L. Tzetlin in the year 1961.It is an automaton with linear tactic $L_{2m,2}$ with two action and with 2m states. m states correspond to each action.

Let the states $\varphi_1, \varphi_2, \varphi_3, \dots, \varphi_m$ corresponds to the first action (for example to be careful agent) and states $\varphi_{m+1}, \varphi_{m+2}, \varphi_{m+3}, \dots, \varphi_{2m}$ corresponds to the second action (for example to be risky agent).

When s(t)=1 (nonpenalty) the change of state of automaton describes by following equations: $\varphi(t+1) = \varphi_{i+1}$ if $\varphi(t) = \varphi_i$ and i=1,2,3,…,m-1,m+1,…,2m-1, $\varphi(t+1) = \varphi_i$ if $\varphi(t) = \varphi_i$ and i=m or i=2m. When s(t)=0 (penalty) the change of state of automaton describes by following equations: $\varphi(t+1) = \varphi_{i-1}$ if $\varphi(t) = \varphi_i$ and i=,2,3,…,m,m+1,…,2m,

$\varphi(t+1) = \varphi_{i+m}$ if $\varphi(t) = \varphi_1$ and $\varphi(t+1) = \varphi_{i-m}$. if $\varphi(t) = \varphi_m$

It is proved in the Thetlin works that this automaton sequences m=1,2,... is asymptotically optimal in stationary random media. There is no stationary random media in the our model but we must note that in mentioned work of Gelfand and Tzetlin some games of automata was investigated. These researches was fulfilled by computer modeling and it was shown by their results that some asymptotic optimal automata can successfully play in some interesting games. Therefore we can consider our model with automata as a participants as some game of automata similar it in the Gelfand, Tzetlin and their coworkers the computer research.

For full description of our model the closed market with automata $L_{2m,2}$ as a participants we must define the probabilities of nonpenalty for all agents in our model(sellers which have sold the all their commodity, buyers which have spent all their money and for other participants of market).

We shall mean the estimation by agent of change of his position on the market as a reaction of random media on the previous action of participant. At first we define the probability of nonpenalty for sellers which had sold anyl quantity of the commodity in the begin of moment t . These sellers have sold his commodity by average price $w_i(t)$ and if $w_i(t) > u(t)$ they became buyers and can buy commodity in moment t+1 by prices which are near u(t). Therefore their estimation of success is $\frac{w_i(t)}{u(t)}$. The probability of nonpenalty will be the ratio of this value to maximal possible value of $w_i(t)/u(t)((v_i(t) + max_{\alpha_i(t)=-1} v_j(t))/(2u(t)))$.

Let when $w_i(t)$ tends to u(t) then probability of nonpenalty tends to 1/2

If $w_i(t) \leq u(t)$ then they become waiting agents which have only money.

The probability of nonpenalty will be the ratio of this value to minimal possible value of $w_i(t)((v_i(t) + u(t-1))/2)$ in this case.

Let when $w_i(t)$ equals to u(t) then probability of nonpenalty equals to ½.

Because according to our algorithm must be $min_{\alpha_i(t)=-1}v_i(t) \geq u(t-1)$ and$max_{\alpha_i(t)=1}v_i$(t)≤ $u(t-1)$ when t>1, we can to establish the following probability of nonpenalty:

$$p_{k_i(t)}=\begin{cases}\frac{1}{2}+\frac{w_i(t)-u(t)}{v_i(t)+max_{\alpha_j(t)=-1}v_j(t)-2u(t)} \text{ if } w_j(t)>u(t)\\ \frac{1}{2} \text{ if } w_i(t)=u(t)\\ \frac{1}{2}-\frac{u(t)-w_i(t)}{2u(t)-u(t-1)-v_i(t)} \text{ if } w_i(t)\leq u(t)\end{cases}$$

When our participant is a buyer, which had spent any part of his money we can use the same but symmetrical arguments and can receive the following probability of nonpenalty.

$$p_{k_i(t)}=\begin{cases}\frac{1}{2}+\frac{w_i(t)-u(t)}{v_i(t)+min_{\alpha_j(t)=1}v_j(t)-2u(t)} \; if \; w_j(t)<u(t)\\ \frac{1}{2} if \; w_i(t)=u(t)\\ \frac{1}{2}-\frac{u(t)-w_i(t)}{2u(t)-u(t-1)-v_i(t)} \; if \; w_i(t)\geq u(t)\end{cases}$$

Now we define the probability of nonpenalty for agents which had some quantity of the commodity in the begin of moment t but they took not part in trade at all. There are many possibilities to define the probabilities of nonpenalty, but we shall use one from simplest definitions of these values for the first stage of the creation and investigation of our system.

Let note that a growth of average price of market is profitable for the agent which has commodity and his probability of penalty or nonpenalty equals 1/2 when u(t)=u(t-1) . When u(t)>u(t-1) then probability of nonpenalty is more then

1/2.When u(t)<u(t-1) then probability of nonpenalty is less then 1/2 . If difference between u(t) and u(t-1) is maximal possible for the state in moment t then the probability equals one. If difference between u(t) and u(t-1) is minimal possible for the state in moment t then the probability equals zero. We define the probability of nonrenalty by using a ratio of change of u(t) in the last moment of time (u(t)-u(t-1)) to maximal possible change of u(u) for spectrum of prices of this market in moment t. We must note that really the situation on the marker (u(t)) not depends from action of such participant. Consequently a probability of nopenalty also not depends on these actions, but we shall write $p_{k_i(t)}$ and shall know that this index means nothing. So the probability of nonpenaly after action k(k=1,-1,careful agent ,risky agent) is given in this case by the formula

$$p_{k_i}(t) = \begin{cases} \frac{1}{2} + \frac{u(t) - u(t-1)}{[max_{\alpha_i(t)=-1} v_i(t) - u(t-1)]} \; if \; u(t) > u(t-1) \\ \frac{1}{2} \; if \; u(t) = u\,(t-1) \\ ½ - \frac{u(t-1) - u(t)}{u(t-1) - min_{\alpha_i(t)=1} v_i(t)} \; if \; u(t) < u(t-1) \end{cases}$$

We define also the probability of nonpenalty for agents which had some quality of the money in the begin of moment t but could not spend all money or they took not part in trade at all. Let note that a decrease of average price of market is profitable for the agent which has money and his probability or nonpenalty equals 1/2 when u(t)=u(t-1) . Let note that a decrease of average price of market is profitable for the agent which has money . So the probability of nonpenaly after action k(k=1,-1) is given in this case by the formula which similar to the similar formula for agent which has commodity :

$$p_{k_i}(t) = \begin{cases} \frac{1}{2} - \frac{u(t)-u(t-1)}{[max_{\alpha_i(t)=-1} v_i(t) - u(t-1)} \; if\; u(t) > u(t-1) \\ \frac{1}{2} \; if\; u(t) = u\;(t-1) \\ ½ + \frac{u(t-1)-u(t)}{u(t-1) - min_{\alpha_i(t)=1} v_i(t)} \; if\; u(t) < u(t-1) \end{cases}$$

We shall further in this section to investigate this market as a media with these probabilities of penalty or nonpenalty and automata $L_{2m,2}$ as participants with different initial condition of market and different memory of the automaton by many computer experiments.

The most important fact which we have received from our computer experiments from model with automats $L_{2m,2}$ as participants is following.

If all participants of market are automata $L_{2m,2}$ then there exists such moment of time τ_1 that for each t> τ_1 takes place r(t)∈ $M_1(d)$, where for all points of set $M_1(d)M$ will be $u_1 - 20d \leq u(t) \leq u_1$+20d. Where $u_1 = (\sum_{t=\tau_1}^{t=\tau_1+\Theta} u(t))$,Θ is enough large.

In the fig 8 we can see three trajectories of u(t) for which u(0) are different but for all of them the previous assertion take place

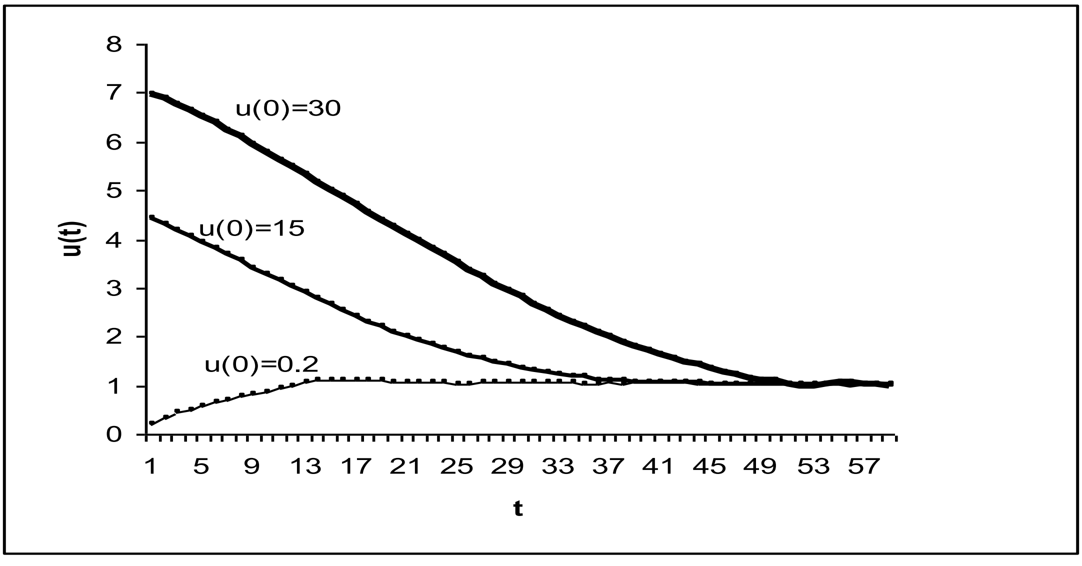

Figure 8. The graphs of u(t) in the cases of different initial conditions, where $\tau_1 > 4500$ and where participants are automata $L_{2m,2}$ (N=300, d=0.005,m=8,u(0)=30,u(0)=15,u(0)=0.2. 0<t<5500).

We can see from fig.8 that for many different initial value u(0) value r(t) (t>τ_1) belong to set M_1 . In this figure we can see the trajectories only for three initial conditions, but we have fulfilled many experiments to be sure in previous assertion. We can see in all these computer experiments that in the set $M_1(d)$ all trajectories hesitate in bounded interval nearly from the average on time value u(t) just the same way as it is in our assertion. The examples of oscillations in the set $M_1(d)$ of same trajectories from fig 8 we see in fig 9.

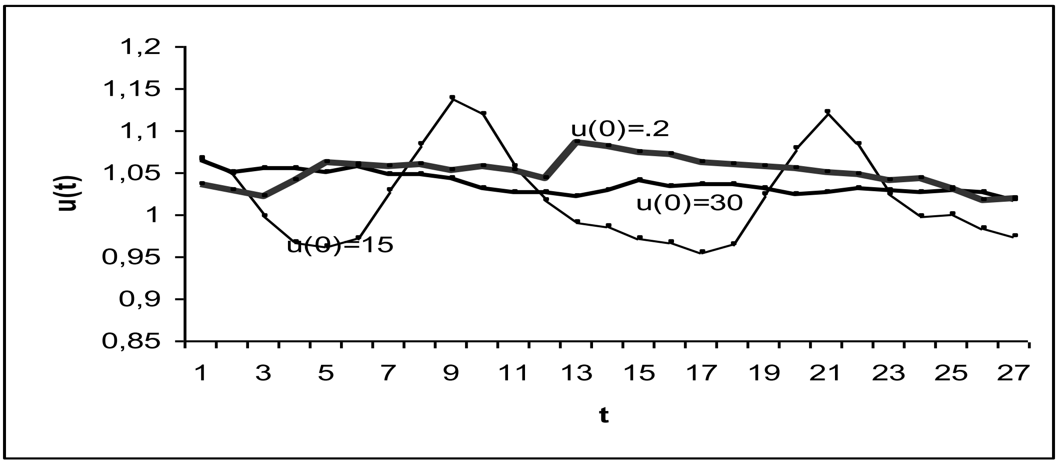

Figure 9 . The graphs of u(t) for 8500<t<10000 for cases of different initial condition but with the same volume of memory of automata $L_{2m,2}$ (N=300, d=0.005,m=8,u(0)=30,u(0)=15,u(0)=0.2. 0<t<5500).

These oscillations have a different character for different initial condition but our assertion is fulfilled for them. We have fulfilled many experiments with different volume of memory of automata from m=1 until m=8. The our assertion is fulfilled in all experiments but trajectories shows different character of their oscillations for different m . We can see it on the fig 9. where we can see irregular oscillation of trajectories but for m=2 the oscillation is similar to regular one. We saw in our experiments that the average value of number of state for k(t)=1,-1 in moment t is

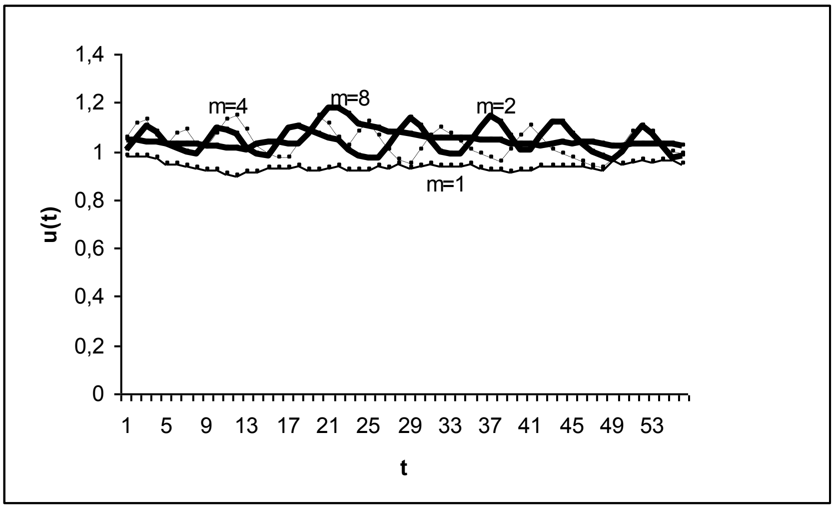

Figure 9. Trajectories of u(t) for 6500<t<8000 for same initial condition and different volume of memory of automata.. (N=300, d=0.005,m=1,2,4,8,u(0)=15,. 6500<t<8000).

more 1 and less 2 . Let note that we said about average value, but real state can be with number m or 2m sometimes.

We investigated in our experiment the ratio of careful agent and risky agent in each moment of time and dynamics of value S(t).

Our computer experiments have shown that the number of careful agent have hesitated around half of number of all participants of market, but sometime was about one quarter or three quarter of N.

The value S(t) had shown very small variation. We think than absence some large change of distribution assets between partners is connected with the fact that all partners have the same mechanism of choice may be. We should like to say that is the first research of similar model, and it will be need to investigate these problems also in further research of this or more adequate model.

6. Conclusion

We have formulated and investigated the simple model of closed market which is similar to stock market at least in few features. Not only the our desire to prove

analytically a some characteristics of a spectrum of prices but also our desire to extract the some features of market which are consequences of logically warranted mechanism of behavior of agents in the case of careful and risky choice. We also have tried to investigate the closed market as a game of automata with linear asymptotical optimal automata as participants. We most note that automata can use both careful and risky choice depend on his state. Automata can also change his state and demonstrate purposeful behavior. There are two central result of investigation of our model by computer experiments. The first is fact that in all experiments the average price of market has a small deviation from his average value along the time(averaging began after enough large moment of time T0) when choices of all participants of market are careful. The second is fact that in all computer experiments the average price of market hesitates near his average value along the time (averaging also began after enough large moment of time T0), with not constant but bounded amplitudes and not constant period when all participants of market use only risky choice. The using the careful and risky choices of one participant of market was investigated in the case the consideration of the dynamics of this market as a game of automata. It is useful to note that behavior of trajectory of average price of market when all participants are careful is not similar to situation on the stock market even in the case of day when external situation changes in the very small way (almost constant fundamental value). But case when choices of all participants of the market are risky is more similar to real behavior of stock market price when the fundamental value) is almost constant during the day.
We ask our readers to understand that this work is first step in study of closed market (stock market, for example) by computer agent based model of type of the game of automata. That is a reason of many lacks of this work and also a reason to discuss of possibility to remove this lacks and the path of further research.
The first we cannot have fulfilled analytical description of the steady set $M_o(d)$ and for model with automata as agents we cannot have fulfilled any analytical investigation of steady set $M_1(d)$ and of the spectrum of prices. The great mathematical difficulties in analytical investigation of a similar systems is the our

justification may be. At least we do not know now are it possible to create some more simple models of interaction of participants of market? But we know that is necessary to investigate some other models of decision making of agents.
In this research we had investigated the market with careful choice of participants and market with risky behavior of participant of market separately and we have united two previous model by using automata. But we use the very simple case of probability of nonpenalty and most simple structure of automaton for the first step of research./
The action of participant (choice of new status and choice of new price) which we named the risky is the action really with a small risk and we need to include in the next similar models more risky action which is more corresponding to real behavior of participant of market (for example overweighting of a small subjective probabilities).By our opinion this can to have as a consequence the trajectories of average price of market more similar to reality .The automaton $L_{2m,2}$ is not unique automaton which was considered in investigations of games of automata by Gelfand , Tzetlin and coworkers. There are also stochastic automata/ So we have large scope for using of different models of a individual behavior of agents. We have a few hopes to successes of analytical investigation of our agent based model of closed market, because we have many singularities as consequence of our mechanisms of market interaction, but there is many hopes to investigation of corresponding markets by computer experiment with new similar models.

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