THE MODEL OF SOCIAL INFLUENCES ON PRICE

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Abstract

The simple model is formulated and investigated in this paper. We try to explain by this model why many successful restaurants, theatres and similar enterprises do not raise prices even in spite of the shortage of places. There is a probability of a choice of one of the restaurants by each consumer. This probability depends on the quantity of the consumers preferring the first restaurant at the previous moment of time and it depends also on relation of the prices of restaurants. Dynamics of distribution of probabilities are described by a uniform Markov chain. It is shown that a stationary distribution exists in the Markov chain and that the chain converges to stationary distribution from any initial distribution. The existence of a equilibrium state is also shown. This is a state when it is not profitable for any of restaurants to change the prices, though almost all consumers prefer only one of restaurants. We have asymmetric preference of consumers in the equilibrium state of the symmetrical system. The preferred restaurant does not increase its price if the system of the two restaurants is in equilibrium. The equilibrium explains (within the framework of our model, of course) the example of restaurants, which was considered by Gary S. Becker.

I. INTRODUCTION

We can see how the different individual economic activities become coordinated as a result of economic interactions. Economic theory says rather little about this. The basic paradigm of the economic theory is the supposition which asserts that each individual takes decisions in isolation, and uses only the information received through some general market signals, such as prices. The standard model does not deny that agents interact but rather that they interact only through the price system. Yet agents do, in fact, trade with each other, communicate with each other and learn from each other.

One sort of interaction is an exchange of information between economic agents. Often one agent receives this information by observation of the behavior of other agents. There are many social and economic situations in which the decision making of participants is influenced by actions of the other participants around him. In decision making individuals sometimes use a combination of their own information and information on the behavior of other individuals; sometimes they use only information on the behavior of other participants. We can see many examples of this phenomenon in economic life: when people have to choose between two restaurants, in formation and change of fashion, in the financial market, et al. There are examples of similar phenomena in zoology, politics and social life. Some authors use the term "herd behavior" to refer to situations in which everyone is doing the same thing others are doing even when their private information suggests doing something quite different. For a long time economists have observed and described similar phenomena. But the creation and the use of herd behavior models to explain some of these phenomena began not very long ago.

Richard Topol presented a model [10], in which he considered the behavior of the investors in a market in which collective opinion exists. He gave a theoretical explanation of the observed movement of stock market prices by using microeconomic processes representing the dynamics of individual behavior.

David S.Scharfstein and Jeremy C.Stein examine [9] some forces that can lead to herd behavior in investment. In some circumstances, a manager mimics the investment decisions of other managers. He assumes that such behavior decreases his responsibility for his decision. This behavior can be rational from the point of view of managers that are concerned about reputation in the labor market. There is only one interaction among the participants: the signals about the state of the system that one participant observes, depends on the actions of the other participants. The authors also discuss the application of their model to the stock market and to corporate investments.

Arthur, David, et al [1], [2] considered a series of economic models that use a stochastic process as a basis. They used these models to explain technological progress.

Abhijit V.Banerjee in his paper [3] considers the model of herd behavior. There is some queue for decision makers. In this model when each decision maker makes his decision, he looks at the decision made by a previous decision maker. Each participant is doing the same that other participants have done but without using his own information. The resulting equilibrium is not effective from a social point of view.

There is some queue of participants in the Sushi Bikhchandani, et al., [5] model of herd behavior. Participants make decisions one after another. Each decision maker takes into account the behavior of the preceding individual without regard to his own information. The information cascades, which arise in their models, are explanation of the herd behavior in politics, zoology, medical practice and finance.

Yehning Chen also considers informational externalities as a cause of a contagious bank run in his paper [6]. His approach is similar to the Sushi Bikhchandani, et al., approach. But Chen uses this approach of decision making of some problems of the financial market and describes the microeconomic conditions of the contagious bank run.

The entomologists observed that a collective of ants in an apparently symmetric situation behave in an asymmetric way. When they faced identical food sources, the ants exploited one of them more intensively than the other. Furthermore, from time to time they switched their attention to the source that they had previously neglected.

To explain some cases of ant behavior, Alan Kirman presents a simple model [7]. Individual behavior is a probabilistic choice of source in moment of time. There is no queue of decision makers in the Kirman model, but the stochastic collisions of participants determines the behavior of decision makers. Information about the preference of another ant (by odors which show the direction of his movement), which the first ant receives when he meets the other ant, affects the probability. This interaction is asymmetrical in the case of ants. There are two external parameters (probabilities, same for all participants), which are being used for choice by the participants. On the whole the dynamic model is a Markov chain. So there is a stationary distribution, which under some values of the parameters corresponds to the ants behavior with switching. The switching in the dynamics of the Kirman model is a consequence of the probabilistic character of the model.

Garry S. Becker considers a very interesting example [4]. When consumers were faced with two apparently very similar restaurants on either side of a street, a large majority chose one rather than the other, even though this involved waiting in a queue. Becker asked: “Why doesn’t the popular restaurant raise prices, which would reduce the queue for a seat but expand profit?” He assumes the pleasure of the goods is a greater factor than the number of people who want to consume it. Thus Becker presuppose that the preference of one consumer depends not only on prices but also on the summary preference of all other consumers (i.e., of the demand). Formally it means that the demand of individual i (d_i) depends on price (p) and total demand (D):

$$d_i = f_i(p, D); \quad p = \sum f_i(p, D) = F(p, D), \quad D = F(p, D)$$

Becker showed that there is some price $p_{\max}$ such that the demand $D(p)$ has a discontinuity in the point $p_{\max}$ and demand drops to zero for any small increase of p in the point $p_{\max}$. But there is some question concerning Becker's assumptions. How can the individual consumer know the total demand at the moment of time when he is prepared to present his own demand?

Becker's restaurant example [4] is interesting in its simplicity, but this example also gives us a rather simple language for the description and investigation of a similar phenomena. This is one of the reasons to formulate and investigate the model of this phenomenon.

We can presuppose that the consumer has no summary information until he meets with one other consumer, similar to the Kirman model. It is the extreme case. A consumer can compare his own preference and the partner's preference. He will choose his own new preference by using information about the two preferences and the ratio of the restaurant's prices. Naturally his partner does the same. It is the distinction between consumers and ants. So the model will be symmetric, and we shall expect the symmetric summary behavior of all consumers. The equilibrium state of the model will most likely also be symmetric. So the model will be symmetric and we shall expect the symmetric summary behavior of all consumers. The equilibrium state of the model will most likely also be symmetric. This does not correspond to empirical data. Therefore, in our model we consider the other extreme case as a consequence of Becker's supposition. Each consumer knows the summary preference of all consumers, i.e., the demand. We formulated and investigated the simple model for explanations of the restaurant paradox.

Section II is devoted to formulation of the model of the system which was investigated in Gary S. Becker [4]. The system consists of two similar restaurants with various prices and populations, which includes the identical consumers. Each consumer makes the decision which restaurant is preferable for him in every moment of time. The probability of choosing the first restaurant (and consequently the probability of choosing the second restaurant) exists for every consumer. This probability depends on the number of consumers who preferred the first restaurant at a previous moment and also on the relation of the prices of both restaurants. The dynamics of the distribution of the probabilities (the probability of having in at this moment of time the given number of consumers who prefer the first restaurant) is described by some uniform Markov chain like that of Kirman's model [7].

In Section III we assume that the prices of the restaurant are constant during the long run. The existence of the stationary distribution and the convergence in the direction of this distribution from any initial distribution, is shown in this section. It is true, that these conclusions are caused by some assumptions about the nature of a probability – the probability of choosing the first restaurant by the given consumer in this moment. The process of the achievement of the stationary preference (the probabilistic distribution of preference of one of the restaurants by the given number of consumers) is a consequence of the interaction of consumers when they choose the preferred restaurant. If the prices of one restaurant are significantly more than the prices of other restaurant then there is only one final steady preference and this preference is the globally steady preference (i.e. this preference obtains from any initial preference). If the prices of both restaurants are roughly equal then there is the set of final steady preferences. The various final preferences obtains for various initial preferences in this case.

Further, in Section IV, we consider the case, when at the reached stationary distribution the restaurants managers can change the prices. We consider a pair of ratios of the prices of the restaurants and a distribution of probabilities that the given number of consumers prefer the first restaurant, as a state of the system. We define a state in which it is not profitable for any of the restaurants to change the prices, though almost all consumers prefer only one of restaurants (it is the stationary distribution of probability) as the steady equilibrium state. It is shown that at least two steady equilibrium states exist in the system at given parameters of the system. All consumers prefer the first restaurant in the first steady equilibrium state, and they prefer the other restaurant in the second steady equilibrium state. The existence of steady equilibrium states is the answer to the question, which was asked by Becker. The popular restaurant does not raise prices because it does not increase its profit, although it would reduce the queue for seats. The price of the preferable restaurant in the steady equilibrium state corresponds to price $p_{\max}$ in the Becker paper. The small increase of this price induces the switching, and the other restaurant becomes the preferable one for all consumers. The perspectives of the further investigation of our model by taking into account some important characteristics of the real behavior of restaurant visitors, or of participants of a market, are discussed in the conclusion of the paper. The comparative value of each group of models mentioned above, is discussed in the conclusion. The perspectives of a development of the models of the herd behavior are also discussed in the conclusion.

II. THE MODEL

There is also some population which consists of N consumers and each consumer chooses the restaurant at each moment of time t (the time is discrete $t = 0,1,2,3,...$) and let $k(t)$ be the number of consumers, which have chosen the first restaurant at moment t.

The average price of the first restaurant is v_1 and the average price of the second restaurant is v_2. We shall consider the $v_1 \le v$, $v_2 \le v$. If $v_1 > v$ and $v_2 > v$, then consumers prefer not to visit both restaurants. If, for example, only $v_1 > v$, then consumer will not visit the first restaurant at any condition. Let $w = v_2 / v_1$.

Denote by $q(w(t), k(t))$ the probability of choose by consumer of the first restaurant in the moment $t+1$.

It is obviously that $q(w,k)$ is defined for $w > 0$, $N \ge k \ge 0$.

We shall assume that $q(w,k)$ is a continuous and nondecreasing function of w at each k.

Thus, we assume that probability for a consumer to choose the first restaurant does not vary or is increasing, when the price of the first restaurant decreases. Probability for a consumer to choose the first restaurant does not vary or falls, when the price of the second restaurant decreases.

Also we assume that probability for a consumer to choose at the moment $t+1$ the first restaurant does not decrease, when the number of consumers, which chose the first restaurant at the moment t, grows, i.e. $q(w,k) \le q(w,k+1)$, $k = 0,1,2,...N-1$.

If k (number of consumers, which have chosen at the moment t the first restaurant) satisfies to a condition $0 \le k \le N$ then there is a ratio of the average prices of restaurants $w_1(k)$ such that $q(w,k) = 0 \quad 0 \le w \le w_1(k)$. It means in particular the following. If at the moment t number of consumers, which have chosen the first restaurant, is equal k and the average price of the second restaurant does not vary, then there is some $v_1(k)$ such that for $v_1 \ge v_1(k)$ the probability for consumer to choose the second restaurant is equal to one. Such assumption is quite natural, because if w is very close to zero then v_1 thus surpasses v_2, that the probability of a choice by the consumer of the first restaurant under all conditions is equal to zero.

Let's assume also that at every k there is ratio of the average prices of restaurants $w_2(k)$ such that for $w \ge w_2(k)$ probability of choice by consumer the first restaurant in the moment *t*+1 is equal to 1, i.e. *q(w,k)*=1 if $0 \le w_2 \le w_2(k) \le w$ 0≤.

Suppose in addition that the function $q(w,k)$ is a increasing

function of w at any fixed k and when $w_1(k) \le w \le w_2(k) : \frac{\partial q(w,k)}{\partial w}$.
We assume that $q(w,k) < q(w,k+1)$ when $w_1(k) \le w \le w_2(k)$ or $w_1(k+1) \le w \le w_2(k+1)$.

These two suppositions mean that if $w_1(k) \le w \le w_2(k)$ or $w_1(k+1) \le w \le w_2(k+1)$ then consumer react upon the change of a ratio of the prices of both restaurants.

It follows from the our suppositions that $w_1(k) \ge w_1(k+1), \quad w_2(k) \ge w_2(k+1)$ and $0 < w_1(k) < w_2(k), \; k = 0,1,2,...N$.

We will do the important supposition now. If all consumers prefer the first restaurant in moment t then each consumer prefers the first restaurant with probability is equal one not only when $v_1(t) \le v_2(t)$ but also for some $v_1(t) > v_2(t)$. This supposition means $w_2(N) < 1$.

So a consumer chooses the first restaurant at $k(t) = N$ even if the average price of the first restaurant is a little more than the average price of the second restaurant. This supposition shows the important role of a social influence on the choice of a consumer and it is very important from economic point of view.

Let us remark that we assumed above the probability of the choice of any restaurant by consumer is equal zero at any values of other parameters, if the average price of this restaurant exceeds some V. Therefore it should more correctly to consider that probability of the choice of the first restaurant by consumer depends on values of restaurants prices $q(v_1, v_2, w, k)$. But $q(v_1, v_2, w, k) = q(w, k)$ when $v_1 \le v$ _ $v_2 \le v$ by our supposition. The choice of only one from two restaurants by consumer will be interesting for us (we consider the case when $v_1 \le v$ _ $v_2 \le v$). Therefore we can consider the probability of choice of the first restaurant by consumer as function of w and k only.

Let's remark that our restaurants are identical and $N - k$ is the number of consumers that prefer the second restaurant therefore the $q(w, k)$ must satisfy to the symmetry condition:

$$q(w,k) + q(w^{-1}, N-k) = 1 \qquad (1)$$

We can show that:

$$w_1(0) = \frac{1}{w_2(N)}, \; w_2(0) = \frac{1}{w_1(N)} \qquad (2)$$

by using the following symmetry conditions:

$$q(w_1(0),0) + q((w_1(0))^{-1}, N) = 1,$$

$$q(w_2(0),0) + q((w_2(0))^{-1}, N) = 1,$$

$$q(w_1(N), N) + q((w_1(N))^{-1}, 0) = 1,$$

$$q(w_2(N), N) + q((w_2(N))^{-1}, 0) = 1.$$

The inequalities $w_2(N) > 1 > w_1(0)$ follow from these symmetry conditions and from the preceding supposition.

We can see disposition of the points $w_1(k),\ w_2(k)$ for $k = 0,1,...N$. On the Fig.1

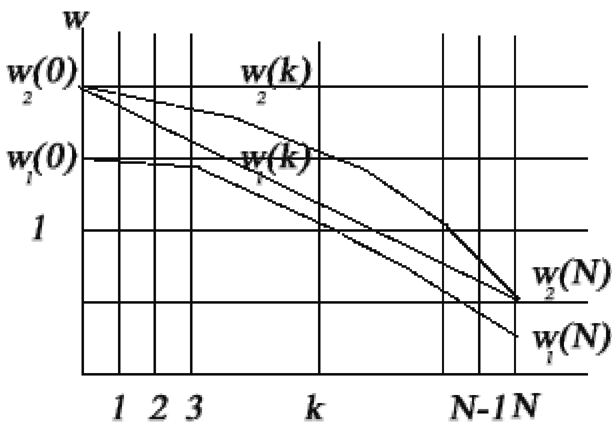

Fig.1

We shall denote further $w_2(N)$ by u and $w_1(N)$ by ω. Then $w_1(0)=1/u$, $w_2(0)=1/\omega$.

We can consider also the probability $r_t(w,k)$ to find $k(t)=k$ at $v_2/v_1=w$, where $r_t(w,k)$ satisfies to the following equality.

$$\sum_{k=0}^{k=N} r_t(w,k) = 1$$

If $k(t)=k$, then the probability of $k(t+1)=l$ is

$$N!/[1!(N-1)!](q(w,k))^l(1-q(w,k))^{N-l} \qquad (3)$$

We shall consider the process during the period when prices are constant, and consequently $w(t)=const$ during the period. The following Markov chain describes our process:

$$r_{t+1}(w,k)=(N!/(N-l)!l!\sum_{k=0}^{K=N}(q(w,k))^l(1-q(w,k))^{N-l}r_t(w,k) \qquad (4)$$

The first question is the following: which distributions of probability are stationary distributions of probability for this Markov chain?

III. STATIONARY DISTRIBUTION

If at some w is $q(w,0)=0$ and $q(w,N)=1$, there are two stationary distributions:

$R_1(k)$: $r(w,0)=1$, $r(w,k)=0\ \forall k\neq 0$;

$R_2(k)$: $r(w,N)=1$, $r(w,k)=0\ \forall k\neq N$. (5)

If $q(w,0)=0$, $q(w,N)=1$ then any convex combination of these probability distributions $\alpha R_1(k)+(1-\alpha)R_2(k)$, $r(w,0)=\alpha$, $r(w,N)=1-\alpha$, $r(w,k)=0\ \forall(k\neq 0$ and $k\neq N)$, $0\leq\alpha\leq 1$ is also stationary probability distribution. Therefore we will name two probability distributions (5) by fundamental probability distributions.

If at some w is $q(w,0)=0$ but $q(w,N)\neq 1$ then there is only one stationary distribution of probability $R_1(k)$: $r(w,0)=1$, $r(w,k)=0\ \forall k\neq 0$.

If at some w is $q(w,N)=0$ but $q(w,0)\neq 0$ then there is only one stationary distribution of probability $R_2(k)$: $r(w,N)=1$, $r(w,k)=0\ \forall k\neq N$.

We shall assume that prices are constant during the some period of the time and that our process (4) attains of some neighborhood of stationary distribution of probability during this period. There is the following assertion.

Assertion 1. If we consider w as a constant in the moments of time t ($t=0,1,2,3\ldots$) then there are the following characteristics of our Markov chain (4):

1. If $w \geq 1/u$ then the probability distribution $r_t(w,k)$ converges to stationary probability distribution $R_1(k)$: $r(w,0)=1$, $r(w,k)=0\ \forall k\neq 0$.

2. If $w\leq u$ then the probability distribution $r_t(w,k)$ converges to stationary probability distribution $R_2(k)$: $r(w,N)=1$, $r(w,k)=0\ \forall k\neq N$.

3. If $u<w<1/u$ then the probability distribution $r_t(w,k)$ converge to stationary probability distribution $\alpha R_1(k)+(1-\alpha)R_2(k)$:$r(w,0)=\alpha, r(w,N)=1-\alpha\ r(w,k)=0\ \forall k\neq 0$ *or* $k\neq N$, $0<\alpha<1$

where α depends on the initial distribution $r_0(w,k)$.

Consequence from assertion 1. There is no other stationary probability distributions of Markov chain (4) except two fundamental stationary probability distributions and their convex combinations.

The proof of the assertion 1. We can write by using (4) the following:

For $w>w_1$ we have $\mu=0$, $0<\nu<N$, and from (7):

$$r_{t+1}(w,0)=(1-q(w,0))^N r_t(w,0)+\sum_{K=1}^{k=N}(1-q(w,k))^N r_t(w,k) \qquad (6)$$

$$r_{t+1}(w,N)=(q(w,N))^N r_t(w,N)+\sum_{K=1}^{k=N}(q(w,k))^N r_t(w,k) \qquad (7)$$

For $w>1/u$ (i.e. $q(w,N)=1$, $q(w,0)>0$) we have from (7):

$$r_{t+1}(w,N)-r_t(w,N)=\sum_{K=0}^{k=N-1}(q(w,k))^N r_t(w,k)>0$$

For $w<u$ (i.e. $q(w,0)=0$, $q(w,N)>0$) we have from (6):

$$r_{t+1}(w,0)-r_t(w,0)=\sum_{K=1}^{k=N}(1-q(w,k))^N r_t(w,k)>0$$

For $u\leq w\leq 1/u$ (i.e. $q(w,0)=0$, $q(w,N)=0$) we have from (6),(7):

$$r_{t+1}(w,N)-r_t(w,N)=\sum_{k=0}^{k=N-1}(q(w,k))^N r_t(w,k)>0 \qquad (8)$$

$$r_{t+1}(w,0)-r_t(w,0)=\sum_{k=1}^{k=N}(1-q(w,k))^N r_t(w,k)>0 \qquad (9)$$

Thus the both our sequences $r_t(w,o)$, $r_t(w,N)$ are increasing and bounded above by 1. Therefore there is limit of those sequences $R(w,0)$, $R(w,N)$. Moreover the right parts of equalities (8),(9) converge to zero. We shall remark that in the right part (8) and(9) cofactor of each $r_t(w,k)$ are nonnegative. If in (8) cofactor at $r_t(w,k)$ equals to zero, then obviously cofactor of the same $r_t(w,k)$ in (9) is positive. Opposite assertion also is correct. Therefore we have: if $1\leq k\leq N$ and $u\leq w$

$\leq 1/u$ then $r_t(w,k) \to 0$ as $t\to\infty$. When $u>w$ or $w>1/u$ then in the right part of the corresponding equalities cofactors of $r_t(w,k)$ are positives. Therefore also $r_t(w,k)\to 0$ as $t\to\infty$.

When $w\geq 1/\omega$, we have from (4):

$$r_{t+1}(w,N)=\sum_{k=0}^{k=N} r_t(w,k)$$

When $w\leq\omega$, we have also from (4):

$$r_{t+1}(w,0)=\sum_{k=0}^{k=N} r_t(w,k)$$

It follows from (2): $r_{t+1}(w,N)=1$, when $w\geq 1/\omega$ and $r_{t+1}(w,0)=1$, when $w\leq\omega$.

The assertion 1 is proved.

The assertion 1 speaks the following.

If the price of the second restaurant surpasses enough the price of the first restaurant, then the probability, that all consumers prefer the first restaurant, converges to unit in course of time at any initial distribution of preferences.

If the price of the first restaurant surpasses enough the price of the second restaurant, then the probability, that all consumers prefer the second restaurant, converges to unit in course of time at any initial distribution of preferences.

If the prices of both restaurants are almost equal, probability, that all consumers prefer the first restaurant, converges to α in course of time and probability, that all consumers prefer the second restaurant, converges to $1-\alpha$ in course of time. Moreover $0\leq\alpha\leq 1$ and the value α depends on the initial distribution of preferences.

IY. THE EQUILIBRIUM STATES

We shall call a complex of pair of a prices v_1, v_2 and some probability distribution $r(k)$ by the state of our system in moment of time t.

We shall call a complex of pair of prices v_1, v_2 and of stationary distribution of probability $r^{(0)}(v_2/v_1,k)=r^{(0)}(w,k)$, which correspond to this pair of prices, by the steady state of our system. The Markov chain (4) at given ratio of prices w converges to a stationary probability distribution $r^{(0)}(w,k)$.

Properly from assertion 1 the unique stationary distribution of probability $r^{(0)}(w,k)$ corresponds to one values of w and the set of stationary distributions of probability $r^{(0)}(w,k)$ corresponds to other values of w.

If $v_2/v_1>1/u$, then only $R_1(k)$ is a such stationary distribution of probability.

If $v_2/v_1<u$, then only $R_2(k)$ is a such stationary distribution of probability.

If $1/u \geq v_2/v_1 \geq u$, then $\alpha R_1(k)+(1-\alpha)R_2(k)$,where $0\leq\alpha\leq 1$ (including $R_1(k)$and $R_2(k)$), are a such stationary distributions of probability.

If a managers of restaurants set the new prices $v_1^{(1)},v_2^{(1)}$ in the state $v_1,v_2,R_1(k)$, where $v_2/v_1\geq u$, then two cases are possible. The previous stationary distribution of probability $R_1(k)$ remains in the case when $v_2^{(1)}/v_1^{(1)}\geq u$. The transition along Markov chain (4) to new stationary distribution of probability begin, and $R_2(k)$ will be the new stationary distribution of probability in the case when $v_2^{(1)}/v_1^{(1)}<u$. Analogous situation will be in the case of state $v_1,v_2,R_1(k)$ where $v_2/v_1\leq 1/u$.

We shall name the switching a transition from the one steady state with fundamental stationary distribution of probability to the other steady state with the other fundamental stationary distribution of probability.

The preference of all consumers turns from the one restaurant toward the other restaurant as a result of the switching.

Let there was the state $v_1,v_2,\alpha R_1(k)+(1-\alpha)R_2(k)$, where $1/u\geq v_2/v_1\geq u$, and let the prices become in the next moment $v_1^{(1)},v_2^{(1)}$ such that $v_2^{(1)}/v_1^{(1)}>1/u$ or $u>v_2^{(1)}/v_1^{(1)}$. In this case we shall have the state $v_1^{(1)},v_2^{(1)},R_1(k)$ or the state $v_1^{(1)},v_2^{(1)},R_2(k)$ after the switching.

Let c be the average expenditure for service of one visitor in each from restaurants.

Let denote by $p_1(t)$ a profit of the first restaurant in the moment t and denote by $p_2(t)$ a profit of the second restaurant in the same moment t. The expected profit of the first restaurant equals in moment of time t

$$Ep_1(t)=(v_1(t)-c)\sum_{k=0}^{N} kr_t(w,k) = (v_1(t)-c)E_t k \,, \tag{10}$$

the expected profit of the first restaurant equals in moment of time t:

$$Ep_2(t)=(v_2(t)-c)\sum_{k=0}^{N} k(1-r_t(w,k)) = (v_2(t)-c)(N-E_t k)\,, \tag{11}$$

where $E_t k=\sum_{k=0}^{N} kr_t(w,k)$

It is naturally to presuppose that a manager of restaurant wants that the profit of his restaurant would be most high. Let assume also that managers know or at least they guess which fundamental stationary probability distribution ($R_1(k)$ or $R_2(k)$) will be reached by $r_t(w,k)$ at $w>1/u$ or $w<u$.

Manager of every restaurant has a two ways to raise the profit of his restaurant. The first way is to set such most high price that the switching not begin yet. The second way is to set such price that the switching begin and the number of visitors of his restaurant will become more after the switching.

For example, if $r_t(w,k)=R_1(k)$ and the manager of the second restaurant does not change the price v_2 then manager of the first restaurant tries to set the price such $v_1^{(1)}=v_2/u$, that $r_t(w,k)=R_1(k)$ will be yet. If the manager of the first restaurant does not change the price v_1 and $r_t(w,k)=R_2(k)$ then manager of the second restaurant tries to set the such price $v_2^{(1)}=v_1/u$ at which would be yet $r_t(w,k)=R_2(k)$.

But at the same time the manager of the restaurant, which is not preferred by consumers, want to attain of the switching (i.e. transition from fundamental stationary distribution of probability $R_1(k)$ to fundamental stationary distribution of probability $R_2(k)$ in the one cases and conversely in the other cases) by decreasing price of restaurant. When the price v_1 not varies and $r_t(w,k)=R_1(k)$ then the manager of the second restaurant can attain of the switching by assignment of the price $v_2^{(1)}<uv1$. When the price v_2 not varies and $r_t(w,k)=R_2(k)$ then the manager of the first restaurant can attain of the switching by assignment of the price $v_1^{(1)}<uv_2$.

Let make one more assumption about behavior of a managers. If the manager of the first restaurant sets the new price v_1^* then the manager of the other restaurant set the price v_2^* such that at once or after switching, which correspond to the new pair of prices v_1^*,v_2^* , the profit of this restaurant will be maximal profit when price of the first restaurant is v_1^*,and price of the second restaurant v_2 varies from 0 until V. The same will be when the second restaurant sets some price v_2^*.

A state of our system, in which any change of a price of every restaurant reduces of the profit of this restaurant at once or after switching (transition from one fundamental stationary distribution of probability to other one), which correspond to a new pair of prices, we name by the equilibrium state.

A state of our system, which is the steady state and equilibrium state simultaneously, we name by the steady equilibrium state of our system.

Let remark, that we mean the equilibrium in some game but not the equilibrium of supply and demand. We shall consider further steady equilibrium states in which a supply is more than a demand.

We shall investigate further in this section the steady equilibrium states of the system of two restaurants and of N consumers.

Let S be the number of places in the first restaurant and of course S be also the number of places in the second restaurant.

Let be $S \geq N$. If we have a steady state with fundamental stationary distribution of probability $R_1(k)$ then the expected profit of the first restaurant equals:

$Ep_1(t)=(v_1(t)-c)N$,

and the expected profit of the second restaurant equals:

$Ep_2(t)=(v_2(t)-c)0=0$.

If we have a steady state with fundamental stationary distribution of probability $R_2(k)$ then:

$Ep_1(t)=(v_1(t)-c)0=0$, $Ep_2(t)=(v_2(t)-c)N$.

There is the following assertion.

Assertion 2. Let $S \geq N$ then there are the steady equilibrium states of our system:

$v_1=c/u>c$, $c \leq v_{2\leq} \leq V$, $R_1(k)$,

and $c \leq v_1 \leq V$, $v_2=c/u>c$, $R_2(k)$.

The proof of the assertion 2. Let consider all states of the system, when $v_1=c/u$, ($u<1$, $v_1>c$) and a distribution of probability that the first restaurant is preferred by k consumers is the fundamental stationary distribution of probability $R_1(k)$. We have $Ep_1=(v_1-c)N>0$, $Ep_2=0$ in these states.

If manager of the second restaurant will set his price v_2 such that $v_2/v_1=v_2/c<u$ then the switching from fundamental distribution $R_1(k)$ to fundamental distribution $R_2(k)$ will begin in the given moment. But it will be in this moment v_2/c, and consequently it will be $Ep_2<0$ i.e. the profit of the second restaurant will reduce. Thus the such change of price of the second restaurant, which induces the switching, reduces the profit of the second restaurant.

The first restaurant can try to increase his profit by increasing his price v_1. But if $v_1^{(1)}>c/u$, then the second restaurant can begin the switching at the price $v_2^{(1)}$, which satisfy to the inequalities $c<c/u<v_2^{(1)}<uv_1^{(1)}$. When the switching will be finished then will be: $Ep_1^{(1)}=(v_1^{(1)}-c)0=0$, $Ep_2^{(1)}=(v_2^{(1)}-c)N>Ep_2=0$. It means that the switching is profitable to the second restaurant. He will effect it, if the first restaurant will increase his price. Thus the increasing of the price by the first restaurant will reduce his profit.

It should be from here that states of the system $v_1=c/u>c$, $c \leq v_2 \leq V$, $R_1(k)$ are steady equilibrium states.

The proof of the fact, that states of the system $c \leq v_1 \leq V$, $v_2=c/u>c$, $R_2(k)$ is steady equilibrium states, is completely symmetric to the previous proof.

The assertion 2 is proved.

The assertion 2 shows that there are two sets of steady equilibrium states. From the one side the transition from one set to other set is connected with negative profit (loss) for one of two restaurants. On the other side after transition to the other fundamental stationary distribution of probability this restaurant can increase his price, may be he can cover his loss and can receive profit after that. Thus overcoming of some “barrier of investment” (it similar to a barrier to entry) is necessary for transition from one set of steady equilibrium states to other set. Besides we must remark that the price of preferred restaurant in the steady equilibrium state is less than the maximal price V because there is risk of beginning of switching by manager of the other restaurant.

Let now be $S<N$ and $2S\geq N$. The similar case of the shortage of places in preferred restaurant was considered by Gary S Becker [4].

If in the moment of time t we have $w\geq u$, $R_1(k)$ then the consumers, which prefer the first restaurant, are divided into two groups. The consumer from first group came in the first restaurant the consumer from the second group could come in the second restaurant only. It is consequences of the shortage of a places. Every consumer came in one of two restaurants, because $2S\geq N$.

The expected profit of the first restaurant equals:

$$Ep_1=(v_1-c)S. \quad (12)$$

The expected profit of the second restaurant equals:

$$Ep_2=(v_2-c)(N-S). \quad (13)$$

If in the given moment we have $w\leq 1/u$ then the expected profit of the first restaurant equals:

$$Ep_1=(v_1-c)(N-S) \quad (12a),$$

the expected profit of the second restaurant equals:

$$Ep_2=(v_2-c)S \quad (13a).$$

Denote $S_1=N(V-c)/(V-c + uV-c)$. It is obviously that $0<<N/2<S_1<N$. There is the following assertion.

Assertion 3. Let $S<N$, $N<2S$ and $u>c/V$.

1) If $S\leq S_1=N(V-c)/(V-c + uV-c)$ (14)

then there are two steady equilibrium states of our system :

$v_1=V$, $v_2=V$, $R_1(k)$ and

2) $v_1=V$, $v_2=V\ R_2(k)$.

3) If $S>S_1=N(V-c)/(V-c+uV-c)$ (15)

then there are two steady equilibrium states of our system:

$v_1=1/u((V-c)(N-S)/S+c)$, $v_2=V$, $R_1(k)$ and

$v_1=V$, $v_2=1/u((V-c)(N-S)+c)$, $R_2(k)$.

The proof of the assertion 3.

1. Let consider the state $v_1=V$, $v_2=V$, $R_1(k)$. In this state $w=1$ and the profits of each restaurant is given by the equalities (12),(13), when $v_1=V$, $v_2=V$: $Ep_1=(V-c)S$, $Ep_2=(V-c)(N-S)$.

The profit of the first restaurant is maximal of possible ones at distribution $R_1(k)$, because his price is maximal. The switching (transition to fundamental stationary distribution of probability $R_2(k)$) reduces his profit, because $S>N-S$.

The manager of the second restaurant can hope to increase profit of the second restaurant by a switching. He must set such price that $w<u$ i.e. $v_2<uV$ $(uv>c, u>c/V)$. When the switching will finish the expected profit of the second restaurant will satisfy to the inequality $Ep_2(1)<(uV-c)S$. We have from (14):

$$S(V-c)+S(uV-c) \leq N(V-c),\ (V-c)(N-S)\geq(uV-c).$$

Therefore $Ep_2^{(1)}\leq Ep_2$.

Thus after the completion of the switching, when the fundamental stationary distribution of probability $R_2(k)$ will be established, the expected profit of the second restaurant will not be more than it was in the initial state. It means that the switching is not profitable for the second restaurant.

So any switching not induces the increasing of the expected profit of each of restaurants. Therefore, if the condition (14) is fulfilled, the state $v_1=V$, $v_2=V$, $R_1(k)$ is a steady equilibrium state of our system.

The proof of the fact, that the state $v_1=V$, $v_2=V$, $R_2(k)$ is a steady equilibrium state, is completely symmetric to the previous proof.

$$S(c/u-c)<(N-S)(V-c)<S(uV-c),\ c/u<v_1<V \qquad (16)$$

The expected profit of each of restaurant is given by equalities (12),(13) at

$$v_1=1/u((V-c)(N-S)/S+\_),\ v_2=V:$$

$$Ep_1=(1/u((V-c)(N-S)/S + c)-c)S=1/u((V-c)(N-S))+(c/u-c)S.,$$

$$Ep_2 =(V-c)(N-S), \qquad (17)$$

$$Ep_1 >Ep_2.$$

We have in this state:

$$w=v_2/v_1=V/(1/u((V-\_)(N-S)/S+c)))>V/(1/u(uV-c+c)=1,\ w= v_2/v_1>1.$$

If the switching will begin at price of the first restaurant $v_1^{(1)}\leq V$, then after completion of this switching will be $Ep_1^{(1)}=((v_1^{(1)}-c)(N-S)\leq(V-c)(N-S)<Ep_1$. The expected profit of the first restaurant not increases after completion of the switching.

The price of the second restaurant is maximal and second restaurant can increase his profit by the switching only.

If the second restaurant reduces his price to attain a switching and if a price of the first restaurant remain equal $v_1=1/u((V-c)(N-S)/S+\_)$, then a new price of the second restaurant must satisfy condition: $v_2(1)<u(1/u((V-c)(N-S)/S+c))=(V-c)(N-S)+c$. But in this case: $Ep_2^{(1)}=(v_2^{(1)}-c)S<((V-c)(N-S)/S)S=(V-c)(N-S) =Ep_2$. The expected profit of the second restaurant reduces as a result of any switching under price of the first restaurant $v_1=1/u((V-c)(N-S)/S+\_)$.

It is necessary finally to consider a case, when the first restaurant increases a price to increase a profit. It means: $v_1^{(1)}>1/u((V-c)(N-S)/S+c)$, but in this case second restaurant can establish the price $v_2^{(1)}$ such, that: $uv_1^{(1)}>v_2^{(1)}>uv_1=(V-c)(N-S)/S+c$. Moreover will be $w^{(1)}=v_2^{(1)}/v_1^{(1)}< v_2^{(1)}/v_1<u$ and a switching will begin. Will be $Ep_1^{(1)}=(v_1^{(1)}-c)(N-S)<(V-c)(N-S)=Ep_1$, $Ep_2^{(1)}>((V-c)(N-S)=Ep_2$ after completion of the switching. It means that the second restaurant is interested in a switching under this price of the first restaurant. Thus the expected profit of the first restaurant reduces and the expected profit of the second restaurant increases as a result of the increasing of the price by the first restaurant. So the reduction of the expected profit of the first restaurant will be result of any increase of his price.

Therefore change of a price by the first restaurant is not profitable for himself, as well as the change of a price by the second restaurant is not profitable for himself. So, the state $v_1=1/u((V-c)(N-S)/S+\_)$, $v_2=V$, $R_1(k)$ is a steady equilibrium state of our system.

The proof of the fact, that the state $v_1=V$, $v_2=1/u((V-c)(N-S)/S+c)$, $R_2(k)$ is a steady equilibrium state, is completely symmetric to the previous proof.

The assertion 3 is proved.

The assertion shows that pairs of the steady equilibrium states, which correspond to fundamental stationary distributions of probability, exist at some conditions. Moreover, the price of the preferred restaurant can be less than the maximal price because the risk of the beginning of the switching by the other restaurant exists.

The switching can give the restaurant the final profit because this restaurant will become preferred. In this case the restaurant can increase its price more than its switching price. But up to achievement of the fundamental stationary distribution of probability, when this restaurant become preferred, this restaurant will bear losses. Thus the transition from the one steady equilibrium state to the other one is connected with the overcoming of some barrier.

The existence of steady equilibrium states, which were described above, and existence of the barrier of the transition among them, explain the paradox, which was considered by Becker.

Actually, if the system is in a steady equilibrium state then a change of the price of any restaurant induces as a final result a reduction of the profit of this restaurant. In particular, under this condition a change of the price by a restaurant which is preferred by consumers under shortage of places for visitors in this restaurant, induces as a final result one of the following situations, either the profit of the preferred restaurant will be reduced or the other restaurant will become preferred.

Unfortunately the assertion 3 not describes all steady equilibrium states. For example it is possible to show that under $S<S_1$ and $S/N<\alpha$ or $S<S_1$ and $\alpha<(N-S)/S$ the states of the set: $v_1=V, v_2=V, \alpha R_1(k)+(1-\alpha)R_2(k)$ are the steady equilibrium states of our system. Analogously it is possible to show that when $S_1<S<N(V-c)/(V-c+u^2V-c)$ and $S/N<\alpha$ all states of the set: $v_1=1/u((V-c)(N-s)/S+c), v_2=V, \alpha R_1(k)+(1-\alpha)R_2(k)$ are steady equilibrium states of our system, also when $S_1<S<N(V-c)/(V-c+u^2V-c)$ and $\alpha<(N-S)/S$ all states of the set: $v_1=V, v_2=1/u((V-c)(N-s)/S+c)$, $\alpha R_1(k)+(1-\alpha)R_2(k)$ are steady equilibrium states of our system

Remark. We assumed in the our consideration that a time for switching is small at any condition. But really the time for switching from the fundamental distribution of probability $R_1(k)$ to the fundamental distribution of probability $R_2(k)$ equals 1 when $w\leq\omega$ and it tends to infinity when w tends to u. Similarly the time for switching from the fundamental distribution of probability $R_2(k)$ to the fundamental distribution of probability $R_1(k)$ equals 1 when $w\geq 1/\omega$ and it tends to infinity when w tends to $1/u$. There is no switching from $R_1(k)$ to $R_2(k)$ or from $R_2(k)$ to $R_1(k)$ when $1/u \geq w \geq u$. Furthermore if will be $w>1/u$ or $w<u$ after the finite time the distribution of probability $r_t(w,k)$ will satisfies to one of two conditions $r_t(w,0)>1-\gamma$ or $r_t(w,N)>1-\gamma$ where γ is small. We did not consider these important properties of our system for simplification of our text. But we can remark that it is possible to give proofs of our assertions with consideration of these properties.

V. CONCLUSION

In our model the probability of choice of the first restaurant by a consumer not depends on time. The probability of having this or that number of consumers choosing the first restaurant (hence, the number of consumers choosing the second restaurant) varies with time. This probability stabilizes after some period of time. If a ratio of the prices is more than some threshold value, or a ratio of prices is less than some threshold value, then, after a sufficiently short time we arrive at a situation, when a majority of consumers with a unit probability prefers one of the restaurants. It is possible to presuppose, that such dynamics of the model is similar to the dynamics of a real consumer choice, although we presupposed that the consumer is very informed. The nature of a stationary distribution of probability of a choice of the first restaurant by the given number of the consumers, depends on a ratio of the prices of the restaurants. It seems to be very similar to a situation, which was considered in Gary S. Becker [4]. If a ratio of the prices is between threshold values then we receive a final distribution of probabilities, at which the majority of the consumers prefers the first restaurant with probability α, and they prefer the second restaurant with probability $1-\alpha$. This situation differs from that considered in [4], and reminds one of Alan Kirman's result [7]. It proved the existence of equilibrium states for the cases when the ratio of prices is more than the threshold values, or the ratio of prices is less than the threshold values. The equilibrium states are the states in which a change of his price is not profitable for any restaurant (nor preferred by the majority of consumers, nor others). If our model in outline reflects the nature of choice of one of two restaurants by consumers, then the existence of equilibrium states is an explanation of the observed phenomenon of the stability of the prices of restaurants in the case of shortage of places in one of them. It is also an answer to the question, which was asked by Becker.

Tastes and preferences of the consumer are represented in standard economic theory by the utility function, and the behavior of the consumer is describing by maximization of the utility function at the limits of financial resource. The classic theory of consumer demand does not consider the influence of one consumer on another consumer nor does it consider the other interactions among consumers. In contrast our models demonstrate that interactions between the given consumer and all other consumers plays an important role.

In the formation and changing of fashion, in the emergence of great popularity of some films and many similar cases, the social factor (e.g., samples of behavior of other consumers affect on the behavior of every consumer) plays an essential role in the formation of a consumer demand. It is possible that the model which was considered above is applicable to research of situations in which social interaction plays a major role in forming a demand. However, the presence not of two species of the goods (two restaurants), but of many species, is an essential factor in most of these cases. Therefore the extension of our model to case M restaurants would be important. The creation of similar models, using our model as a base, is not complicated. For every complex of prices, and possibly some complexes of other parameters, the degree of preference, or some preference ordering of the restaurants or goods arises in this case instead of the one preferable and the other unpreferable restaurant. But analysis and investigation of the model of M restaurant is far from simple. Perhaps it is possible to define the notion of equilibrium in this case and to investigate at least some equilibrium states.

Besides the question about constancy of prices Garry Becker asks other questions in his paper [4]. He asks, "If Price is not raised when demand exceeds supply, why does not output expand to close the gap?”

Becker proposes two possible answers to this question. The first answer is the following: Customers are fickle and a booming business is very fragile, therefore the restaurant is afraid to make an additional investment in increasing the quantity of places. The restaurant assumes that increasing the quantity of places in some conditions can lead to a sharp fall of the demand and even to bankruptcy. Another answer to the question why the supply is not increasing is the following: aggregate demand depends not only on price and aggregate demand, but also positively on the gap between demand and supply.

The first answer corresponds to the results of our model. Let us consider expected profit of the first restaurant in an equilibrium steady state as a function of quantity of places in restaurant $Ep_1(S)$. Then maximal value of this function is in point $S=S_1=N(V-c)/(V-c+uV-c)$. If the quantity of places in the restaurant equals S_1 then it is not profitable for the preferred restaurant to increase the quantity of places. Apropos, it follows from assertion 3 that increasing his price is also not profitable for the preferred restaurant.

The second answer, which was proposed by Becker, is not packed in the framework of our model. But this presupposition likely corresponds to the real behavior and it provides a topic for further investigation, including modification of our model. For example taking account of the dependence of our probability q from *S/N* (level of shortage when the first restaurant is preferred – index of prestige of the visit of this restaurant) is most interesting in the case $N>S$, $k=N$. When there are shortages and queues, then a consumer must expend time to stand in a queue. He will take into consideration the time, which he must expend in a queue and the probability of being seated in the preferred restaurant.

Other suppositions about the dynamics of processes of stabilization of consumer choice may be interesting. For example the period of time which is necessary to attain steady states can turn out to be less, if we presuppose that the probability of choice of the first restaurant by a consumer also depends on the increment of the quantity of consumers who prefer the first restaurant.

Alan Kirman[8] emphasized the importance of models, in which agents only interact with a limited subset of other agents, their neighbors. He writes that economy is a complex adaptive system, that the very process of learning and adaptation, and the feedback from the consequence of that adaptation, generate highly complicated dynamics.

Every consumer in our model knows the demand, i.e. the summary preference of all consumers. One of the assumptions of Garry Becker consists in this. Maybe the consumer has a great deal of information about the state of the system. It is interesting to consider the model, in which the volume of information about preferences of other consumers, which every consumer uses, is significantly less. For example consider a consumer who does not know the volume of demand, and let him meet with one other consumer. Our consumer finds out the preferences of his partner in this moment as a result of their meeting. He can then compare his preference and preference of his partner. He shall choose his new preference using the information of the two preferences and ratio of prices of restaurants w. His partner will do the same and the process continues until stabilization occurs. The process is symmetric, but the final state or final steady equilibrium state of new model will be asymmetric (as in our model).

It is possible to formulate the models in which the participants make their decision one after another by queue, and each participant uses information about the behavior of preceding participants and imitates the behavior of those participants. These models are similar to models described in the introduction and are more realistic and important to understanding the processes which take place in economy. The above proposed modification of our models in which the choices of a participant does not depend on the summary choice of participants, but only on the choices of the participants which he meets, is a step on the way to joining the two groups of models.

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