THE DYNAMICAL MODELS OF ONE GOOD MARKET WITH MANY PARTICIPANTS AND THE CHARACTER OF THE PROCESS OF SETTING OF PRICES

Joint French-Russian Symposium, CEPREMAP, Paris, France, 1991

M.M.Voronovitsky

There is one model of exchange betweens suplier and buyer which represent the price of exchange as $\frac{v+w}{2}$, where v is price of suplier and w is price of byuer. That model is usual in microeconomical theory, but the theory discribes only two partisipants case and seldom market with many sellers and buyers.Furthemore microecomical theory not discribes the uneqilibrium states and proces of reaching of the equilibrium.

In the paper a dinamic model of exchange is disscussed where the participants are identical automata. A structure of the automata is found at which the sistem reashes an equilibrium.

According to the theory of economic equilibrum behaviour of an individual participant is supposed to be rational, i.e. it is determined by his wish to maximize his function of usefulness.

This is one of the causes of the situation, that the models of market, considered in this theory are statical, i.e. they don't reflect adaptation characteristics of behaviour of the participants and do not allow us to research the process of setting of the equilibrum. Another path for such a research is given by us in the simple models of single-good market, considered by us before. Their behaviour is not rational, but the algorithm of behaviour in different external market conditions is given.

In short, the participant is an apparatus with a price (requested or offered) - the main variable value. This way of research allowes us to give the formal description of the participant's interaction.

For the biult dynamic model there exists (as proved in previous works) the stationery equilibrumd state that is globally equilibrumd (see [1]-[4]). This allowes us to suppose that this model (at least, quantitatively) describes the real process of setting of the equilibrum in the market.

For the present state of real economy, being characterized with the transition from rationing to market mechanisms of distribution, the most important thing is the analysis of the process of establishment of the equilibrum in different mechanisms of conduct and interaction. We suppose, that the dynamic model of the market, created by us, can be used for such an analysis. In particular, our formal description allowes us to analyze the dynamics of establishment of equilibrum by means of experiments and computerized models.

This work is dedicated to the examination of these questions.As preliminary, we have to describe the main elements of our market model.

At first, we will describe the mechanism of exchange. Let the seller, having x units of goods and ready to sell them at the price, not lower than v, and buyer with u units of money,ready to pay the price,not higher,than w,are meeting at the given moment of time. This means, that, if the seller will be proposed to sell his goods at the price, lower than v, then he refuses from the bargain. The same happens, if the buyer will be asked about the price, higher than w.

It is supposed, that buyer and seller agree only on the price of the goods. For simplifying the model, the volume of sales and purchases is defined only by x and u and the price of exchange. The exchange takes place at the price, equal to $\frac{v+w}{2}$. The exchange is possible, if the condition $v \leq w$ is being fulfilled.

Let's suppose, that as the result of the bargain either the seller will sell all of his goods, or the buyer will spent all of his money, i.e. let $w \geq v$ and ω is the volume of goods sold. Then $\omega = x$ if $x(v+w)/2 \leq u$ and $\omega = \frac{2u}{(v+w)}$ if $(v+w)/2 > u$.

Suppose, that we have m sellers of a good and n buyers of this good. v, x-lowest price and the volume of goods of seller $i, (i = 1, 2, ..., m); w, u$-maximal price and amount of money of buyer j.

For definition, at the given moment of time:

$$v_1 < v_2 < ... < v_m$$

$$w_1 > w_2 > ... > w_n$$

Suppose, that each of the sellers knows the maximal prices of all of the buyers, i.e.$w_1, w_2, ..., w_n$, then it will be natural for him to offer his good to the buyer with the highest w (i. e. to try to sell it at the maximal price). Similarily, each of the buyers knows the minimal prices of all of the sellers' goods, i.e. $v_1, v_2, ..., v_m$ and he will request the seller with the smallest v (to buy the goods at the lowest price). Thus, each of the sellers is choosing the buyer, who is ready to pay the maximal price, and the buyer tries to connect the seller, requesting the minimal price.

We imagine, that this order of exchange approximately describes the logic of the real exchange. If the wishes of the buyer and the seller coincide, and they still possess their goods or money, the exchange takes place in line with rules, described above. If, after the exchange, the seller still has a part of his goods, then he offers it to the buyer at the price ofthe next higher level (w), etc. The buyer requires the good (if he still has any money) of the next lower value v, etc. If $v \leq w$ then the first act of exchange will take place between the seller with minimal v and the buyer with the maximal w. Thus, the process of exchange at the given moment of time consists of several acts of mutual exchange inpairs, and it will finish in the following cases:Either for all of the pairs "seller-buyer", possessing the money and the goods, V will be higher, than W, or the sellers will have no goods or the buyers will have no money at all.For the formal description of the process of exchange we will mark the amount of goods, sold by seller i to buyer j by, w-the price of exchange, ν_i - is the volume of goods, sold by a seller i; y_i - money, gained by him.

Similarily, z_j - the volume of goods, bought by buyer j; μ_j - money, spent by him for the purchase of this good.

The rules of distribution of goods and money between the sellers and buyers with similar corresponding maximal and minimal prices are introduced. For example, all the money, received by the sellers, are first distributed between all of the sellers in equal portions,because the price of the bargain is equal for everyone (as if equal portions of goods are taken from all of the sellers). But the sellers with smaller volumes of goods do not have enough of them for satisfaction of the demand, so the money, that is not spent before, is summed and distributed in similar portions between the sellers, who still have their goods, etc., until all of the money is distributed.

The above-shown models of single-good market are dynamic, i.e. all the quantities, introduced above, depend upon time, which considers to be discrete, i.e.$t = 1, 2, 3...,$, as mentioned above. Each period of time consists of three stages.

During the first stage the exchange of goods and money takes place on the rules,mentioned above.

During the second stage every seller receives the quantty of goods $a_i(t)$, irrespective of time,and every buyer receives such a sum of money $b_j(t)$ (for example it possible $a_i(t) = a_i, b_j(t) = b_j$), that

$$x_i(t+1) = x_i(t) - \nu(t) + a_i(t)$$

$$u_j(t+1) = u_j(t) - \mu(t) + b_j(t) \tag{1}$$

It is supposed, that a and b satisfy the some conditions

1. If a_i and b_j $(i = 1, 2, ..., m : j = 1, 2, ..., n)$ are constants:

$$\sum_{i=1}^{m} a_i = 1, \sum_{j=1}^{m} a_j = 1$$

$$b_1 \geq b_2 \geq ... \geq b_n$$

$$b_j > \gamma/2, b_1 > \frac{1}{n} + \frac{\gamma}{2} \tag{2}$$

2. In the other case:

$$a_i(t) = \frac{y_i(t)}{\sum_{i=1}^{m} y_i(t)}$$

$$b_j(t) = \frac{z_j(t)}{\sum_{j=1}^{n} z_j(t)} i = 1, 2, ...m, j = 1, 2, ...n \tag{3}$$

whereγ is the minimal amount of money distributed. During the third stage the prices of the exchange are changing.

We will examine the rules of changes of maximal and minimal prices of the following kind:

$$v_i(t+1) = v_i(t) + \gamma F_\epsilon(x_i(t), \nu_i(t), x_i(t+1) - x_i(t))\Psi(v_i(t), \frac{y_i(t)}{\nu_i(t)}, F_\epsilon)$$

$$w_j(t+1) = w_j(t) - \gamma F_\epsilon(u_j(t), \mu_j(t), u_j(t+1) - u_j(t))\Phi(w_j(t), \frac{\mu_j(t)}{z_j(t)}, F_\epsilon)$$

The function F_ϵ,available within ± 1 is the "main function",and Ψ and Φ ,having the value of 0 or 1,are the "functions of carefulness".

$$F_\epsilon = \begin{cases} 1 & \text{if } x_i - \epsilon \leq \nu_i \\ -1 & \text{if } 0 \leq \nu_i \leq x_i - \epsilon, x_i(t+1) - x_i(t) \geq 0 \\ 1 & \text{if } 0 \leq \nu_i \leq x_i\epsilon, x_i(t+1) - x_i(t) < 0 \end{cases} \tag{4}$$

Similarily,

$$F_\epsilon = \begin{cases} 1 & \text{if } u_j - \epsilon \leq \mu_j \\ -1 & \text{if } 0 \leq \mu_j \leq u_j - \epsilon, u_j(t+1) - u_j(t) \geq 0 \\ 1 & \text{if } 0 \leq \mu_i \leq u_j - \epsilon, u_j(t+1) - u_j(t) < 0 \end{cases} \tag{5}$$

The sence of the main function F_ϵ is the following. The seller increases the minimal price on ,if he has managed to sell his goods at the t moment. If only a part of his goods was sold,then he increases the price on the value of ,if at the beginning of $t+1$ moment he has got less goods than he had at the t moment as a result of the receival of a. He decreases the price in the opposite situation.

If he has spent just a part of his money,then he decreases his price on the value of , if by the beginning of t+1 moment as the result of the receival of b , he got less money,than it was at the beginning of t period.Let's mark,that the seller or the buyer may not take part in the exchange in the t period because of too big or too small size of maximal or minimal price.In this case they have to increase or decrease their prices due to (6) and (7).

During the increase or deduction of prices every participant of the exchange must follow several limitations: 1.All the prices must be higher than 0. 2.Besides that,the carelessness of the participant is the basis of their memory limitation.That means that every participant of the exchange tries to protect himself from the loss of a big amount of exchange links,that have been existing in the previous period.The functions of carelessness Ψ and Φ look like:

$$\Psi = \begin{cases} 0 & \text{if} v_i = 0, F_\epsilon = -1 \\ 0 & \text{if} v_i + \frac{\gamma}{2} \leq \frac{y_i}{\nu_i}, F_\epsilon = 1 \\ 1 & \text{in the other cases} \end{cases} \tag{6}$$

$$\Phi = \begin{cases} 0 & \text{if} w_j = 0, F_\epsilon = 1 \\ 0 & \text{if} w_j - \frac{\gamma}{2} \leq \frac{\mu_j}{z_j}, F_\epsilon = 1 \\ 1 & \text{in the other cases} \end{cases} \tag{7}$$

Thanks toΨ the seller does not increase his minimal price (as needed according to F_ϵ), if his minimal price V at the moment t differs from average price of the bargain at the t moment for not more than $\gamma/2$ or, if the last buyer,with whom he deals during the t moment has the maximal price w, equal to his minimal v, because without following these rules he may look for another buyer, whose changes of prices are not known to him at the present moment.

At the same time,according to ϕ function the buyer does not decrease his maximal price w (although this is required by F_ϵ), if his maximal price w of the t moment differs form the average bargain price for not more than $\gamma/2$ or at the t moment the last of the sellers, with whom he deals, the minimal price v is equal to maximal w.

Vector with components $x_1(t), x_2(t), ...x_m(t), v_1(t), v_2(t), ..., v_m(t), u_1(t), u_2(t), ..., , u_n(t)$, $w_1(t), w_2(t), ...w_n(t)$ will be mentioned below as $r(t)$-state of the system. Vector $r(t)$ is stationery,if

$$r(t+1) = r(t) = r \tag{7}$$

The state, in which all the money are spent and all the goods are sold, i.e.

$$X(t) = Z(t), U(t) = Y(t)$$

$$X(t) = \sum_{i=1}^{m} x_i(t), U(t) = \sum_{j=1}^{n} u_j(t), Y(t) = \sum_{i=1}^{m} y_i(t), Z(t) = \sum_{j=1}^{n} z_j(t) \tag{8}$$

Let's remark that, if at the t moment the market is in the equilibrumd state $r(t)$ than $r(t+1)$ may not be equilibrumd. So we are interested in the state of the system, which are simultaneously equilibrumd and stationery.

The following facts are available for this model. The first of all is the one theorem on setting the prises. Note:

$$\rho = \max_j w_j(t) - \min_i v_i(t)$$

$$\sigma = min_j w_j(t) - max_i v_i(t)0$$

$$\lambda = \max_j w_j(t) + \min_i v_i(t)/2 \tag{9}$$

Theorem 1. In every initial state r^0 of the sistem there exist such τ, that

$$\rho \leq 2\gamma,$$

$$\sigma \geq 0$$

$$\text{for } t \geq \tau$$

For the case of a_i, b_j are constant (2) there is theorem.

Theorem 1.1 For the equilibrum of r^0 state the following is necessary and significant condition exists:

$$v_i(t) = v(t), w_j(t) = w(t), X(t) = Y(t) = Z(t) = U(t) = 1$$

Theorem 2.1 In every initial state of $r(0)$ system there exists such T ,that $r(t) = r^0, t > T$,where r^0 is the stationery equilibrumd state.

Another mechanism of distribution of entering goods and money between the sellers and buyers according to (3) is considered. Note M_ϵ the set of a stacionary states of the sistem.

Theorem 1.2 For the $r \in M_\epsilon$ it is nesessary and sufficiently:

$$w_j = v_i = 1, i = 1, 2, ..., m, j = 1, 2, ..., n$$

and

$$X = 1, \max_j (u_j - \mu_j) \leq \epsilon$$

or

$$U = 1, \max_i (x_i - \nu_i) \leq \epsilon$$

Theorem 2.2 In every initial state of r^0 system there exists such T ,that for $t > T$ we have $r(t) \in M_\gamma$ whereM_γ is determined by following conditions:

$$0 \leq \sigma \leq \rho \leq 2\gamma$$

$$| 1 - \lambda | \leq \frac{3}{2}\gamma$$

$$| 1 - X | \leq 2(3m + 4)\gamma$$

$$| 1 - U | \leq 2(3n + 4)\gamma$$

For this model the bancrupcy of the participants is possible, i.e. for some sellers the moment comes, that for all $t >$ it will be, and for some buyers the moment comes, that for every it will be, where accuracy of calculations. With that, if at the initial time m sellers and buyers take part in the market, then, as proved by the experiments, since the moment only sellers and buyers take part in the market( ). Thus, for every of represented models of sellers 'and buyers' behaviour the theorems about the existence of the equilibrumd stationery market and about the global equilibrum of this. The last phrase means, that in every initial state the market in due time (depending upon the state) reaches the equilibrum. For a non-perfect market with sufficiency of communication between the participants (this market model is realized during the transition period) the character of the process of setting of the equilibrumd state. If the model of the participant's activity is one of (6)-(7) type, (this process was described in the previous works) then it is proved that the process of setting of the equilibrumd state consists of the following stages: narrowing of the difference of the prices of sellers and buyers, setting of the price near to the level of equilibrum, setting of the equilibrumd flows of goods and money.

Literature.

[1] M.M.Voronovitsky, "On One Exchange Model." Automatika i Telemechanika, 8, 1974 (russian).

[2] M .M. Voronovitsky, "On a Simple Mechanisms of the Interactions a Suplier and a Customer." Economics and Mathematical Methods" XIII, 2, 1977 (russian).

[3] M .M. Voronovitsky, "The Exchange Model at the Least Interaction betweens the Participants." Economics and Mathematcal Methods" XYIII, 5, 1982 (russian).

[4] M .M. Voronovitsky, "On Convergence of the Spectrun of a Prices for the one Class of a Exchange Models." Economics and Mathematcal Methods." Central Economical and Mathematical Institut, Moscow, 1984 (russian).